DOCUMENT RESUME

ED 075 240 24 SE 016 069

TITLE	Self-Paced Physics, Segments 11-14.
INSTITUTION	Naval Academy, Annapolis, Md.; New York Inst. of Tech., Old Westbury.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-8-0446
PUB DATE	70
CONTRACT	N00600-68-C-0749
NOTE	115p.
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	College Science; Instructional Materials; *Mechanics (Physics); Physics; *Problem Sets; Problem Solving; *Remedial Programs; Science Education; *Self Help Programs
IDENTIFIERS	Self Paced Instruction

ABSTRACT

Four segments of the Self-Paced Physics Course materials are presented in this problems and solutions book for use as the third part of student course work. The subject-matter topics are related to impulses, inelastic and elastic collisions, two-dimensional collision problems, universal constant of gravitation, gravitational acceleration and fields, planet weights, gravitational potential energy, and escape speeds. Contained in each segment are information panels, core problems enclosed in a box, core-primed questions, scrambled problem solutions, and true-false questions. A study guide is provided for each segment, and used to answer the true-false questions and to reveal directions for reaching solutions when the correct answer is chosen. When the core problems is answered incorrectly, the study guide requires the student to follcw the remedial or enabling loop, leading, in turn, to the solution of core-primed questions. Also included is an answer sheet for the four segments. (Related documents are SE 016 065 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

FILMED FROM BEST AVAILABLE COPY

DEVELOPED AND PRODUCED UNDER THE UNITED STATES NAVY CONTRACT #N00600-68C-0749 AND UNITED STATES OFFICE OF EDUCATION, BUREAU OF RESEARCH, PROJECT #8-0446, FOR THE UNITED STATES NAVAL ACADEMY

NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY, LONG ISLAND, NEW YORK 11568

Course Materials - Fall, 1970

U.S. DEPARTMENT OF HEALTH EDUCATION & WELFARE OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY

ED 075240

UNITED STATES NAVAL ACADEMY

Self-Paced

PHYSICS

SEGMENTS 11-14

SE 016 069

DEVELOPED AND PRODUCED UNDER THE
U.S. OFFICE OF EDUCATION, BUREAU OF RESEARCH,
PROJECT #8-0446, FOR THE U.S. NAVAL ACADEMY
AT ANNAPOLIS. CONTRACT #N00600-68C-0749.

NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY,

INFORMATION PANEL The Meaning of Impulse

OBJECTIVE

To define the impulse of a force and use this definition in the solution of basic impulse problems.

According to the staff expert of a popular ballistics magazine, a given bullet (0.22 cal long-rifle, 40 gm) develops the following muzzle velocities:

fired from a rifle, 24 inch barrel	1,290 ft/sec
fired from a pistol, 6 inch barrel	420 ft/sec

The initial explosive force of the powder charge is the same in each instance. Yet the bullet is given a much greater momentum by the rifle. Why?

To keep things simple, we'll start by assuming that the identical powder charge will provide the same constant force for propelling the bullet in either the rifle or pistol barrel. The difference in results for the two weapons is clearly a matter of the time interval over which the force acts, the bullet being subjected to the propulsive force for a substantially longer time in the rifle. We say in this case that the rifle provides a larger *impulse* (not a larger force!) than the pistol and we define the magnitude of the impulse for these simple conditions as

$$\text{Impulse} = J = F\Delta t$$

where F is the constant force on the bullet and Δt is the time the force acts. Clearly, a given impulse due to a large force acting for a short time can be duplicated for a smaller force acting for an appropriately longer time.

If the force varies with time, we could then say that the magnitude of the impulse is given by

$$J = \overline{F}\Delta t$$

in which $\overline{F}$ is the average force over the time interval Δt. This immediately leads to the expression

$$J = \int_{t_1}^{t_2} \vec{F}\, dt$$

where t_1 is the time at the beginning of the interval and t_2 is the time at the end of the interval. A rigorous definition of impulse may

next page

continued

then be written:

> *The impulse of a force is the integral of the force over the time interval during which the force acts.*

The magnitude of impulse is therefore the area under the force-time curve.

Impulse has the dimensions of force times time and may be expressed in nt-sec or lb-sec.

You should be prepared to solve problems which require that you

(a) write a general expression for the impulse of a force that is directly proportional to time;

(b) determine impulse magnitude from the area under a given force-time curve;

(c) calculate time of application of a force, given the mass of the object to which it is applied, the magnitude of the impulse, and the average magnitude of the force.

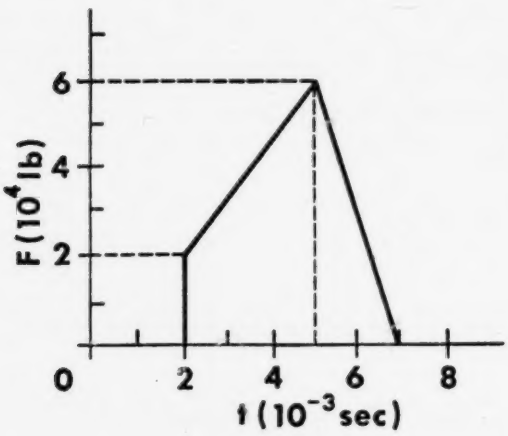
PROBLEMS

1. An impulsive force proportional to time is applied to a block. The constant of proportionality is k, i.e., $F = kt$, and the total time during which the force is applied is T. Assume that at time $t = 0$, $F = 0$. What is the magnitude of the tital impulse?

A. $(1/2)kT^2$
B. FT
C. kT^2
D. $(1/2)(T^2/k)$

2. A 0.5-kg croquet ball is struck by a mallet, receiving an impulse of magnitude 4.5 kg-m/sec. The magnitude of the average force during the time of contact between mallet and ball is 1500 nt. For how long are the two objects in contact?

3.



The accompanying diagram shows the magnitude of the force exerted on an object as a function of time. Assume a constant direction for the force and determine the magnitude of the impulse.

4. An impulsive force proportional to the square of time is applied to a block. The constant of proportionality is k, i.e., $F = kt^2$, and the total time during which the force is applied is T. Assume that at time t = 0, F = 0. What is the magnitude of the total impulse?

A. kT^2
B. $(1/3)kT^2$
C. $(1/3)kT^3$
D. $(1/6)kT^3$

INFORMATION PANEL Impulse and Momentum

OBJECTIVE

To solve problems in which the relationship of impulse to change of momentum is involved.

Newton's second law in momentum terms may be written:

$$\vec{F} = \frac{d\vec{p}}{dt} \quad (1)$$

or

$$d\vec{p} = \vec{F}\,dt \quad (2)$$

in which $\vec{p}$ = momentum, $\vec{F}$ = force, and dt is the time interval over which the force acts.

A body having an initial momentum $\vec{p}_1$ and a final momentum $\vec{p}_2$ undergoes a change of momentum $\vec{p}_2 - \vec{p}_1$ over an interval from t_1 to t_2. The change of momentum is expressed:

$$\vec{p}_2 - \vec{p}_1 = \int_{t_1}^{t_2} d\vec{p} \quad (3)$$

Combining equation (2) and equation (3):

$$\vec{p}_2 - \vec{p}_1 = \int_{t_1}^{t_2} \vec{F}\,dt \quad (4)$$

But since $\int_{t_1}^{t_2} \vec{F}\,dt$ is the impulse $\vec{J}$ of the force $\vec{F}$, then:

$$\vec{p}_2 - \vec{p}_1 = \vec{J} = \Delta\vec{p} \quad (5)$$

or verbally

The change in momentum ($\Delta\vec{p}$) of a body is equal to the impulse (J) of the force which produces the change.

Impulse is a vector quantity expressed in nt-sec or lb-sec. An alternate way to write $\vec{J} = \Delta\vec{p}$ is

$$\vec{F}\Delta t = m\Delta\vec{v} \quad (6)$$

which is probably the most useful form for problems in which the force is constant or can be stated as an average value.

next page

continued

Another useful relationship expresses the connection between the work done on a body and the impulse of a conservative force. Suppose a body of mass m initially at rest ($K_i = 0$; $p_i = 0$) is subjected to an impulse of magnitude J and we want to find the work done. Starting with the work energy theorem:

$W = \Delta K$ but since $K_i = 0$

then $W = K_f$

now $K_f = \frac{1}{2} m v_f^2$ However $p = m v_f$

so $p^2 = m^2 v_f^2$

thus $K_f = \frac{1}{2} \frac{p_f^2}{m}$

so finally $K_f = \frac{1}{2} \frac{J^2}{m}$ since $p_f = J$

You are expected to be able to

(a) determine impulse, given the change of velocity of a given mass, and vice versa;

(b) calculate work done, given the change of momentum or impulse acting on a specified mass.

5. A baseball is thrown by a pitcher at 90 mi/hr toward the strike zone. The batter hits a line drive, reversing the original direction of the ball's motion. If the ball weighs 4 oz, is in contact with the bat for 0.01 sec, and leaves the bat at 150 mi/hr, what is the magnitude of the average force on the ball during the time of contact?

6. A thumbtack cannot be pushed into a piece of hard wood by thumb pressure alone but a few taps with a light-weight hammer can do the job.

Select the statement that best explains why this is true.

A. The surface of a hammer is much harder than the surface of a thumb.

B. Momentum is conserved in the blows of the hammer, but is not conserved by the force of the thumb.

C. The force applied by the hammer is very large compared to the force applied by the thumb.

D. The head of a hammer is more massive than a thumb.

7. A mass of 10 kg moving with a velocity of 8 m/sec is given an impulsive blow such that its velocity is increased to 10 m/sec in the original direction of motion. What is the magnitude of the impulse?

8. A body has a momentum of 50 kg-m/sec. It is given an impulsive blow J = 120 nt-sec at right angles to the direction of its initial momentum. What is the magnitude of the body's new momentum?

9. A 0.5-kg croquet ball at rest is struck by a mallet, receiving an impulse of magnitude 4.5 kg-m/sec. Calculate the work done on the ball by the mallet.

A. 4.5 j
B. 13.5×10^{-3} nt-m
C. 6750 j
D. 20.3 nt-m

10. A rubber ball thrown vertically downward hits the floor with a speed of 40 ft/sec and rebounds with the speed of 30 ft/sec. If the ball weighs 4 oz and is in contact with the floor for 0.01 sec, what is the magnitude of the average force on the ball during the time of contact?

INFORMATION PANEL Conservation of Momentum

OBJECTIVE

To apply the principle of conservation of linear momentum to problems involving interacting bodies.

The principle of conservation of linear momentum may be stated as follows:

> *When the resultant external force acting on a system is zero, the total momentum of the system remains constant in magnitude and direction.*

or

$$\text{when } \vec{F}_{ext} = 0, \text{ then } \vec{P} = \text{constant}$$

Another very useful way to write the symbolic equivalent of the statement of the principle is:

$$\text{when } \vec{F}_{ext} = 0, \text{ then } \Delta\vec{P} = 0$$

which merely emphasizes the fact that a constant momentum does not change, i.e., $\Delta\vec{P} = 0$.

Note that the statement does not contain any qualifications; momentum is *always* conserved in *any* kind of interaction between masses. Unlike energy, momentum is a purely mechanical entity* involving only mass and velocity, so that when it seems to vanish from one part of a system during an interaction, it must appear somewhere else in mechanical form in the same system. Energy, on the other hand, can take many forms--heat, sound, light, etc.--so that the purely mechanical energy of a moving mass may actually disappear from the mechanical system during an internal interaction. Its reappearance in some other form is not always readily apparent but, in any case, the *mechanical energy* of any real system of interacting macroscopic masses is never conserved. Some of the initial mechanical energy is always dissipated in some other form which tends to leave the system. In a number of the problems you will meet, however, you will be asked to assume mechanical energy conservation as a good first approximation.

*Momentum is carried by electromagnetic radiation and other fields. This interpretation of momentum is not used in this course.

next page

continued

In this section, you will be presented with problems that call for

(a) the determination of the final speed of one or both bodies in an interacting two-body system;

(b) the ratio of the kinetic energies of a shell and a recoiling gun.

11. A machine gunner on the bow of a boat fires his gun horizontally. The gun is firing 600 rounds per minute. Each shell weighs 2 ounces and has a muzzle speed of 3200 ft/sec. The combined weight of the boat, gunner, machine gun, etc., is one ton. Neglecting friction and assuming the boat to be initially at rest, what is its speed after five seconds of continuous firing?

A. 80 ft/sec
B. 10 ft/sec
C. 5 ft/sec
D. 2 ft/sec

12. For an isolated system of particles not subjected to a net external force,

A. the kinetic energy remains unchanged
B. the momentum of the individual objects remains the same
C. no collisions are possible if all the particles in the system have the same speed
D. the total momentum of the system is constant

13. A shell has mass m and is fired with a muzzle speed v from a gun of mass M. Assuming a constant acceleration and a recoil speed v_r which is small compared to v, find the expression for the ratio of the kinetic energy of the recoiling gun to the kinetic energy of the shell.

A. v_r/v

B. $(v_r/v)^2$

C. v/v_r

D. $(v/v_r)^2$

14. A man standing on a surface of smooth ice (assumed to be frictionless) fires a 12-gm bullet from a rifle, giving the bullet a muzzle velocity of 600 m/sec. If the combined mass of the man and the gun is 60 kg, calculate the speed with which the man will be set into motion.

INFORMATION PANEL — Inelastic Collisions

OBJECTIVE

To recognize that momentum but not kinetic energy is conserved when two bodies collide inelastically; to solve problems of this type using the principle of conservation of momentum.

An inelastic collision always results in the conversion of some of the kinetic energy of the one or more components into energy of some other form. Thus, kinetic energy is not conserved and we may not use the energy principle in solving inelastic-collision problems. But since momentum is always conserved, this conservation principle may be freely applied.

A convenient and workable way to write the principle of conservation of momentum for two bodies that collide inelastically is:

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

in which m = mass, u = velocity before collision, and v = velocity after collision. The subscripts apply to bodies 1 and 2, respectively. (When the collision is in one dimension, the vector notation may be omitted.)

If the colliding masses become coupled after an inelastic collision, then $v_1 = v_2$ and the conservation equation becomes:

$$m_1\vec{u}_1 + m_2\vec{u}_2 = \vec{v}(m_1 + m_2)$$

where v is the common velocity of the two masses.

Furthermore, if the two masses are equal, the subscripts may be omitted from the "m's" so that we can write

$$u_1 + u_2 = v_1 + v_2 \qquad \text{(do not couple)}$$

$$u_1 + u_2 = 2v \qquad \text{(couple)}$$

You will find that the amount of time consumed in solving inelastic-collision problems can be reduced by using these shortened forms where they are applicable.

next page

continued

This section contains problems in which coupling of colliding masses occurs--that is, purely inelastic collisions.

15. A railroad car of mass 1000 kg is rolling down a track at 3 m/sec. It strikes a stationary car of mass 2000 kg. If the two cars couple together, what is the speed of the combination after the collision? (neglect friction)

A. 1 m/sec
B. 0 m/sec
C. 3 m/sec
D. 2 m/sec

16. Which of the following statements characterizes inelastic collisions?

A. Total momentum and total kinetic energy are conserved.
B. Total momentum is conserved while total kinetic energy is not.
C. Neither total kinetic energy nor total momentum is conserved.
D. Either one or both of the particles involved in the collision reverses its direction.

17. A mass m_1 = 3 kg moves with a speed of 12 m/sec toward a stationary mass m_2 = 9 kg on a table. The two masses collide and stick together. Neglecting friction, find the kinetic energy of the combined body.

A. 848 j
B. 216 j
C. 54 j
D. 27 kg

INFORMATION PANEL Elastic Collisions

OBJECTIVE

To apply the principles of conservation of momentum and energy to the solution of elastic-collision problems.

No one has ever succeeded in setting up a perfectly elastic collision between macroscopic objects in the laboratory. Real objects always experience a heating effect during a collision, largely as the result of intramolecular friction during compression at the instant of impact. The consequent rise of temperature is indicative of the presence of heat energy where none existed before; some of the kinetic energy has undergone conversion to unrecoverable thermal energy, hence kinetic energy has not been conserved.

Practically, however, certain collisions approach the requirements for complete elasticity so closely that conservation of kinetic energy can be assumed without significant error. High-grade steel ball bearings rolling on a plate glass surface exemplify this; so do ordinary billiard balls on a felt topped table, but less closely. In this part of our work, we assume perfect elasticity for such collisions so that we may apply both conservation principles in problem work.

The basic applicable equations are:

(conservation of momentum) $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

(conservation of kinetic energy) $\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$

where m = mass; u = velocity before collision; v = velocity after collision and subscripts refer to body 1 and body 2.

The relationships can be extensively simplified for special cases where the masses are equal or integral multiples of one another.

In this section you should anticipate problems in which you are to

(a) find the velocities of both masses after a perfectly elastic collision;

(b) differentiate between the characteristics of elastic and inelastic collisions.

18. A steel ball of 2-kg mass (m_1), moving to the right along a horizontal frictionless surface at a speed of 40 m/sec, collides head-on with another 2-kg steel ball (m_2) moving to the left at 20 m/sec. After the collision, m_1 recoils and moves to the left at 20 m/sec. Assuming the collision to be perfectly elastic, in which direction and with what speed will m_2 move after the collision?

19. In an elastic collision between two bodies, as contrasted with an inelastic collision,

A. the total momentum is conserved but kinetic energy is not
B. the total kinetic energy is conserved
C. the objects involved in the collisions are constructed of elastic materials such as rubber
D. neither total kinetic energy nor total momentum is conserved

20. Two railroad cars, car 1 and car 2, are rolling on the same track in the same direction. They collide but do not couple together. Which of the following equations correctly describes the relationship between the momenta of the cars? (u's and v's correspond to initial and final velocities)

A. $(1/2)m_1u_1^2 + (1/2)m_2u_2^2 = (1/2)m_1v_1^2 + (1/2)m_2v_2^2$

B. $m_1u_1 + m_2u_2 = (m_1 + m_2)\ v_1;\quad v_1 = v_2$

C. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

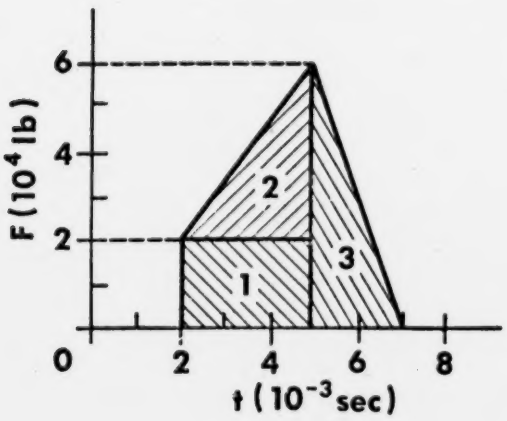
D. $m_1u_1 + m_2u_2 = -(m_1v_1 + m_2v_2)$

21. Two railroad cars collide. Car two has twice the mass of car one, and they are travelling with the same speed in opposite directions. The collision is neither perfectly elastic nor perfectly inelastic. Car one rebounds in the opposite direction with half its initial speed. Find the final velocity of car two in terms of its initial velocity (u_2).

A. $-(1/4)u_2$
B. $(1/4)u_2$
C. $(1/2)u_2$
D. $-(1/2)u_2$

22. Consider a one-dimensional elastic collision between two objects of equal mass ($m_1 = m_2 = 2$ kg). m_1 is moving to the right with a speed of 12 m/sec and collides elastically with the stationary mass m_2. Find the speed of mass m_1 after the collision.

[a] CORRECT ANSWER: 180 lb-sec



The impulse is given by

$$J = \int_{t_1}^{t_2} \vec{F}\, dt$$

which is just the area under the force-time curve. (This expression is equivalent to

$$J = \bar{F}\Delta t$$

where $\bar{F}$ is the time averaged force and Δt is the duration of application.) We break up this area into one rectangular and two triangular segments (marked 1, 2, and 3, respectively). Thus, the area under the force-time curve becomes

$$\text{Area} = (2 \times 10^4)(3 \times 10^{-3}) + \frac{1}{2}(4 \times 10^4)(3 \times 10^{-3}) + \frac{1}{2}(6 \times 10^4)(2 \times 10^{-3})$$

$$= 180 \text{ lb-sec}$$

[b] CORRECT ANSWER: B

The momentum equation is

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

When the relations $m_2 = 2m_1$, $u_1 = -u_2$, and $v_1 = -(1/2)u_1 = (1/2)u_2$ are substituted in (1) we obtain

$$-m_1u_2 + 2m_1u_2 = m_1\frac{u_2}{2} + 2m_1v_2$$

and

$$v_2 = \frac{u_2}{2}\left[-1 + 2 - \frac{1}{2}\right] = \frac{u_2}{4}$$

[a] CORRECT ANSWER: C

The given data are $m_1 = 3$ kg, $m_2 = 9$ kg, $u_1 = 12$ m/sec, and $u_2 = 0$. Since this is an inelastic collision, only the momentum of the system is conserved. The equation for the conservation of momentum is:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

Since the two masses stick together after collision, we can write:

$$v_1 = v_2$$

Thus, equation (1) reduces to

$$v_1 = v_2 = \frac{m_1}{(m_1 + m_2)} u_1 \qquad (2)$$

The kinetic energy of the combined body may now be easily calculated. Thus,

$$KE = \frac{1}{2}(m_1 + m_2)\, v_1^2 = \frac{1}{2}(m_1 + m_2)\frac{m_1^2u_1^2}{(m_1 + m_2)^2} = \frac{1}{2}\frac{m_1^2u_1^2}{(m_1 + m_2)}$$

Substitution of the given data:

$$KE = \frac{1}{2}\frac{(3\text{ kg})^2(12\text{ m/sec})^2}{(3\text{ kg} + 9\text{ kg})} = 54\text{ j}$$

TRUE OR FALSE? Kinetic energy and momentum are both conserved in this problem.

[b] CORRECT ANSWER: D

The work done by the mallet is equal to the increase in kinetic energy of the ball (assuming that the forces involved are conservative). We assume the ball was initially at rest ($K_i = 0$; $P_i = 0$), so $W = \Delta K = K_f$. But

$$K_f = \frac{1}{2}mv_f^2 = \frac{P_f^2}{2m} = \frac{J^2}{2m} = \frac{(4.5)^2}{2(0.5)} = 20.3\text{ nt-m}$$

Thus,

$$W = 20.3\text{ nt-m} = 20.3\text{ j}$$

[a] CORRECT ANSWER: A

The given data are $m_1 = 1000$ kg, $m_2 = 2000$ kg, $u_1 = 3$ m/sec, and $u_2 = 0$. Since there is an inelastic collision, only the momentum is conserved.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

and since the two cars couple together, $v_1 = v_2$. Combining this information, equation (1) becomes

$$m_1u_1 = (m_1 + m_2)\, v_1 \qquad (2)$$

and

$$v_1 = v_2 = \frac{m_1}{m_1 + m_2}\, u_1$$

Substitution of the given data gives us the result

$$v_1 = v_2 = \frac{(1000 \text{ kg})(3 \text{ m/sec})}{(1000 \text{ kg} + 2000 \text{ kg})} = 1 \text{ m/sec}$$

TRUE OR FALSE? The collision described above is to be considered perfectly elastic.

[b] CORRECT ANSWER: B

While momentum is conserved in each and every collision, whether elastic or not, it is only elastic collisions that conserve the kinetic energy as well.

Energy, in general, is always conserved. However, in inelastic collisions some, or all, of the kinetic energy of the objects involved is converted into heat (thermal energy), and sometimes light and sound. The other forms of energy cannot be converted back into mechanical energy--at least not in their entirety. Hence, it is correct to state that in inelastic collisions kinetic energy is lost.

A collision in which all of the kinetic energy is conserved is frequently termed "perfectly elastic." The term "perfectly inelastic" is reserved for collisions in which the (two) bodies involved stick together after the collision.

A - INCORRECT - Total momentum is conserved in all collisions, whether they are elastic or not.

next page

continued

D - INCORRECT - Momentum is conserved in all collisions, whether they are elastic or not. There is something else, however, that distinguishes elastic from inelastic collisions. What is it?

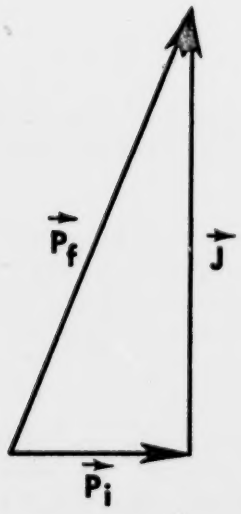
C - INCORRECT - The definition of elastic and inelastic collisions rests purely upon physical phenomena.

In considering whether or not this choice of answer is correct, one must make the distinction between the use of a word in the context of physics, and the use of a word in the context of one's everyday informal speech.

While materials such as rubber are commonly called "elastic" materials, this does not mean that when they enter into collisions, these collisions are elastic.

[a] CORRECT ANSWER: 130 kg-m/sec

The diagram below depicts the situation of this problem. Since we are only interested in the magnitude of $\vec{J}$, we may assign arbitrary directions to $\vec{P}_i$ and $\vec{J}$, as long as these two vectors are at right angles to each other. Now,



$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

or

$$\vec{p}_f = \vec{J} + \vec{p}_i$$

Since $\vec{J}$ is normal to $\vec{p}$ we have

$$p_f = \sqrt{J^2 + p_i^2} = \sqrt{(120)^2 + (50)^2}$$

$$= 130 \text{ kg-m/sec}$$

Note that the units given for J, nt-sec, are the same as the usual MKS units for momentum; namely,

$$(1 \text{ nt}) \times (1 \text{ sec}) = (1 \text{ kg-m/sec}^2) \times (1 \text{ sec}) = 1 \text{ kg-m/sec}$$

[a] CORRECT ANSWER: 12 m/sec

[The given data are $m_1 = 2$ kg, $m_2 = 2$ kg, $u_1 = 12$ m/sec, and $u_2 = 0$.]

Since this is an elastic collision, total kinetic energy as well as total momentum is conserved. The conservation equations are:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

and

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad (2)$$

Substituting $m_1 = m_2$ and $u_2 = 0$ in equations (1) and (2), we get

$$u_1 = v_1 + v_2 \qquad u_1 - v_1 = v_2 \qquad (3)$$

and

$$u_1^2 = v_1^2 + v_2^2 \qquad u_1^2 - v_1^2 = v_2^2 \qquad (4)$$

Dividing equation (4) by equation (3) we obtain

$$u_1 + v_1 = v_2 \qquad (5)$$

Solving equation (3) and equation (5) simultaneously yields:

$$v_1 = 0 \qquad \text{and} \qquad u_1 = v_2$$

Note that this is a general result for a one dimensional elastic collision between equal masses; the velocities of the colliding particles are exchanged, thus $v_2 = 12$ m/sec.

TRUE OR FALSE? In the above solution it has been assumed that kinetic energy is conserved.

[b] CORRECT ANSWER: C

The magnitude of the force is given by $F(t) = kt^2$. Therefore, the magnitude of the impulse is

$$J = \int_0^T F\,dt = \int_0^T (kt^2)\,dt = \frac{1}{3}kT^3$$

TRUE OR FALSE? In this problem, the magnitude of the impulse is constant over the time interval ΔT.

[a] CORRECT ANSWER: D

For a system of two particles, the total change in momentum is zero,

$$\Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

In other words, the total momentum is constant. The extension to any number of particles undergoing collisions is obvious.

A - INCORRECT - For an isolated system of particles, the total kinetic energy of the particles can change. For example, suppose we throw two putty balls of equal mass at each other from opposite directions with the same speed. Just before they collide they will have some kinetic energy, but after the collision they will have been joined together to form a single mass with zero velocity. Thus, after the collision they will have no kinetic energy. Hence, kinetic energy is not conserved.

B - INCORRECT - Your choice of this answer shows that you are not giving as much thought to this question as you ought to. It should be clear to you by now that when the individual particles of a system enter into collisions, there may be changes in the velocities of these particles; hence, there will be changes in the *momenta* of the individual particles of the system.

C - INCORRECT - If all the particles of a system had the same velocity, then there could be no collisions. The particles would all be moving with the same speed and in the same direction. Thus, no one particle could catch up with any other particle to collide with it. However, this answer states that the particles have the same *speed*. There are no qualifications as to direction. Therefore, particles traveling oppositely or obliquely to each other certainly can collide.

[b] CORRECT ANSWER: B

All real collisions on the macroscopic scale are inelastic collisions. While many collisions may be treated as though they were elastic in approximate solutions, there remains a large class that must be treated as inelastic. In solving collision problems involving macroscopic objects, assume that the collisions are inelastic unless otherwise indicated. In inelastic collisions, only momentum is conserved since much of the kinetic energy initially present is converted to other forms such as heat, sound, etc. Mechanical energy is conserved only in a *perfectly elastic collision.*

[a] CORRECT ANSWER: C

The reason we can drive tacks into wood with a hammer and not with our thumb is because of the comparatively large impulsive force applied by the hammer. Let us take an example.

On the average a man can supply about 400 nt of force with his thumb for a short while. Now, we calculate the force applied by a 1-lb hammer having a mass of about 0.5 kg. From the impulse-momentum equation

$$\vec{J} = \Delta\vec{p} = \bar{\vec{F}}\Delta t = m\Delta\vec{v}$$

so

$$\bar{\vec{F}} = \frac{m\Delta\vec{v}}{\Delta t}$$

Suppose that in a light blow the hammer moves with a downward velocity of 15 m/sec (v_i = -15 m/sec), and that it bounces off the tack with a (final) velocity v_f = 5 m/sec. The change in velocity is (written in scalar form)

$$\Delta v = v_f - v_i = 5 - (-15) = 20 \text{ m/sec}$$

If we assume that the hammer and tack are in contact for 0.001 sec, the average force applied on the tack is,

$$\bar{F} = \frac{m\Delta v}{\Delta t} = \frac{(0.5)(20)}{0.001} = 10{,}000 \text{ nt}$$

which is 25 times as great as the force applied by the thumb.

[b] CORRECT ANSWER: 3×10^{-3} sec

Impulse is the product of the average force exerted during a time interval and the time interval. Denoting the average force as $\bar{F}$ and the time interval as Δt, the impulse J is given by

$$J = \bar{F}\Delta t$$

Therefore,

$$\Delta t = J/\bar{F} = \frac{4.5 \text{ kg-m/sec}}{1500 \text{ nt}} \times \frac{\text{nt}}{\text{kg-m/sec}^2} = 3.0 \times 10^{-3} \text{ sec}$$

[a] CORRECT ANSWER: -12 cm/sec

In the absence of a net external force the total momentum of the system must remain constant, in this case, zero, because initially the whole system was at rest.

Therefore, $$M\vec{V} + m\vec{u} = 0 \qquad (1)$$

where M = mass of man with gun = 60 kg
m = mass of the bullet = 12 gm
$\vec{u}$ = velocity of the bullet
$\vec{V}$ = velocity of the man with gun after the gun has been fired

Rearrangement of terms in equation (1) and subsequently, substitutions of numerical values gives

$$\vec{V} = -\frac{m}{M}\vec{u} = -\frac{12 \times 10^{-3}\ \text{kg}}{60\ \text{kg}} \times 600\ \frac{\text{m}}{\text{sec}} = -120 \times 10^{-3}\ \text{m/sec}$$

The man moves with a speed of 12 cm/sec in a direction opposite to that of the bullet.

TRUE OR FALSE? The direction in which the man is accelerated is indicated by the negative sign in front of 120×10^{-3} m/sec.

[b] CORRECT ANSWER: 20 kg-m/sec

Impulse is equal to the change in momentum, $J = \Delta p$. For constant mass,

$$\Delta p = mv_f - mv_o$$
$$= m(v - v_o)$$
$$= m\Delta v$$

Here Δv = 2 m/sec, so the magnitude of the impulse is

$$J = m\Delta v = (10\ \text{kg})(2\ \text{m/sec}) = 20\ \text{kg-m/sec}$$

[a] CORRECT ANSWER: 40 m/sec toward the right

Since this is an elastic collision, both momentum and kinetic energy are conserved. The conservation equations are

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

$$\frac{1}{2}\,m_1u_1^2 + \frac{1}{2}\,m_2u_2^2 = \frac{1}{2}\,m_1v_1^2 + \frac{1}{2}\,m_2v_2^2 \qquad (2)$$

Noting that $m_1 = m_2$, we may rewrite equations (1) and (2) as

$$u_1 - v_1 = v_2 - u_2 \qquad (3)$$

$$u_1^2 - v_1^2 = v_2^2 - u_2^2 \qquad (4)$$

Combining equation (4) with equation (3), we have

$$u_1 + v_1 = v_2 + u_2 \qquad (5)$$

Solving (5) and (3) simultaneously, we obtain

$$u_1 = v_2 \qquad \text{and} \qquad u_2 = v_1$$

These results state that the particles simply exchange velocities during the elastic collision.

Therefore, v_2 = 40 m/sec toward the right.

TRUE OR FALSE? Kinetic energy and momentum are both conserved in this problem.

[b] CORRECT ANSWER: A

The magnitude of the force is given by $F(t) = kt$. Therefore, the magnitude of the impulse is

$$J = \int_0^T F\,dt = \int_0^T (kt)\,dt = \frac{1}{2}\,kT^2$$

TRUE OR FALSE? If the time during which the force is allowed to act is doubled, the magnitude of the impulse would increase to twice its former value.

[a] CORRECT ANSWER: B

Two equally valid ways of viewing this problem will be presented below. In both cases it will be assumed that the total weight of the fired shells is negligible when compared with the total weight of the boat and its contents. We shall justify this assumption shortly.

In the first method we shall use conservation of momentum. In the absence of a net external force the total momentum of the system must remain constant; in this case, zero. Therefore, each time a shell is fired the momentum of the boat (by boat we mean boat plus its contents) will change by an amount equal to the change in the momentum of the shell. Let -v be the velocity of the "boat" relative to the water. (The direction of the fired shells is taken to be positive.) Then, the velocities of each shell before and after firing are respectively -v and -v + u, where u is the muzzle velocity. The change in each shell's velocity, therefore, is

$$\Delta v_\alpha = (-v + u) - (-v) = u \qquad (1)$$

and the change in the shell's momentum is

$$\Delta p_\alpha = m\Delta v = mu \qquad (2)$$

The corresponding change in the boat's momentum is

$$\Delta P_\alpha = -\Delta p_\alpha = -mu \qquad (3)$$

Since each time a shell is fired the momentum of the boat changes in the same direction, the total change in the boat's momentum is the sum of the individual changes. Thus

$$\Delta P = \sum_{\alpha=1}^{n} \Delta P_\alpha = n\Delta P_\alpha = -nmu \qquad (4)$$

where n is the total number of shells fired. The final velocity of the boat is computed from

$$\Delta v = v_f - v_i = v_f - 0 = \frac{\Delta P}{M} = -\frac{nmu}{M} \qquad (5)$$

M being the mass of the boat.

The total number of shells fired is

$$n = 600 \frac{\text{shells}}{\text{min}} \times \frac{1}{60} \frac{\text{min}}{\text{sec}} \times 5 \text{ sec} = 50 \qquad (6)$$

and the mass of each shell is

$$m = \frac{2}{16} \text{ lb} \times \frac{1}{32} \frac{\text{slug}}{\text{lb}} = \frac{2}{16 \times 32} \text{ slug} \qquad (7)$$

next page

continued

The mass of the boat is

$$M = 1 \text{ ton} \times 2000 \frac{\text{lb}}{\text{ton}} \times \frac{1}{32} \frac{\text{slug}}{\text{lb}} = \frac{2000}{32} \text{ slug} \qquad (8)$$

Comparing (7) to (8) we see that our assumption, that the weight (mass) of the shells is negligible to that of the boat, is quite valid. Substituting (6), (7) and (8), along with u = 3200 ft/sec, into (5) we obtain

$$v_f = -\frac{50 \times 2 \times 3200 \times 32}{16 \times 32 \times 2000} = -10 \text{ ft/sec} \qquad (9)$$

The second method involves using the variable mass (rocket) equation. The assumption is that the equation can be used even though the loss of mass is not continuous. This assumption is quite logical since the shells are fired at a steady rapid rate.

In the absence of a net external force equation we can write:

$$u \frac{dM}{dt} = M \frac{dv}{dt} \qquad (10)$$

and

$$u \, dM = M \, dv \qquad (11)$$

This equation can be integrated to give

$$\int_0^{v_f} dv = u \int_{M_i}^{M_f} \frac{dM}{M}$$

or

$$v_f = u \ln\left[\frac{M_f}{M_i}\right] = u \ln\left[\frac{M - nm}{M}\right] = u \ln\left[1 - \frac{nm}{M}\right] \qquad (12)$$

with the symbols having the same meaning as in (5).

Using (6), (7) and (8) we find

$$\frac{nm}{M} = \frac{50 \times 2 \times 32}{16 \times 32 \times 2000} = \frac{1}{320}$$

So (12) becomes

$$v_f = 3200 \ln\left[1 - \frac{1}{320}\right] = 3200 \ln\left[\frac{319}{320}\right] = -10 \text{ ft/sec} \qquad (13)$$

next page

continued

We could, of course, make the assumption that M is constant and then integrate (11) to obtain

$$M\int_0^{v_f} dv = u\int_{M_i}^{M_f} dM$$

or

$$Mv_f = u(M_f - M_i) = u[(M_i - nm) - M_i] = -nmu$$

so

$$v_f = -\frac{nmu}{M}$$

the same as in (5).

TRUE OR FALSE? At any time during the five-second firing period, the total momentum of the system is zero.

[a] CORRECT ANSWER: 54.7 lb

From the impulse-momentum theorem, we have

$$\int_{t_1}^{t_2} \vec{F}\, dt = \Delta\vec{p}$$

or $$\bar{\vec{F}}\Delta t = m\Delta\vec{v}$$

and $$\bar{\vec{F}} = m\frac{\Delta\vec{v}}{\Delta t}$$

But $$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = -30 \text{ ft/sec} - 40 \text{ ft/sec} = -70 \text{ ft/sec}$$

which shows that the change in velocity is in the direction of the rebound.

On substitution of the given data, we obtain

$$\bar{F} = \frac{4}{16} \text{ lb} \times \frac{1}{32 \text{ ft/sec}^2} \times 70 \frac{\text{ft}}{\text{sec}} \times \frac{1}{0.01 \text{ sec}} = 54.7 \text{ lb}$$

TRUE OR FALSE? The initial velocity of impact of the rubber ball on the floor is taken as a negative quantity in the above solution.

[a] CORRECT ANSWER: 275 lb

From the impulse-momentum theorem, we have

$$\int_{t_1}^{t_2} \vec{F}\, dt = \Delta\vec{p}$$

or

$$\bar{\vec{F}}\Delta t = m\Delta\vec{v}$$

and

$$\bar{\vec{F}} = m\,\frac{\Delta\vec{v}}{\Delta t}$$

Now

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 240 \text{ mi/hr}$$

in the direction of the line drive.

Using the given data, we obtain

$$\bar{F} = \frac{4}{16} \text{ lb} \times \frac{1}{32 \text{ ft/sec}^2} \times 240 \times \frac{88}{60} \text{ ft/sec} \times \frac{1}{0.01 \text{ sec}} = 275 \text{ lb}$$

TRUE OR FALSE? This solution is based upon the fact that the y-component of the momentum of the system is zero before and after the collision.

[b] CORRECT ANSWER: C

The equation states that the total momentum before collision is equal to the total momentum after collision, which is true for all collisions in the absence of a net external force.

[a] CORRECT ANSWER: A

From

$$p = mv$$

and

$$K = \frac{1}{2} mv^2$$

we find that

$$K = \frac{p^2}{2m}$$

Using the subscripts s and r for shell and gun, respectively, we have

$$\frac{K_r}{K_s} = \frac{p_r^2/2M}{p_s^2/2m} = \frac{mp_r^2}{Mp_s^2}$$

But from conservation of momentum we know that $\vec{p}_r = -\vec{p}_s$, or $p_r^2 = p_s^2$, so

$$\frac{K_r}{K_s} = \frac{m}{M}$$

Since $p_s = p_r$

$$\frac{m}{M} = \frac{v_r}{v}$$

We find

$$\frac{K_r}{K_s} = \frac{\cancel{m}}{\cancel{m}(v/v_r)} = \frac{v_r}{v}$$

UNITED STATES NAVAL ACADEMY

Self-Paced

Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Collision Problems - One Dimension

OBJECTIVE

To develop a practical, logical pattern for solving collision problems in one dimension by working out a series of typical problems involving two objects.

This Segment consists of drill problems on collisions of various types. In the first portion of it, you are required to solve both elastic and inelastic collision problems in one dimension. A brief review of the basic procedures follows:

ELASTIC COLLISIONS - ONE DIMENSION:

(a) One-dimensional problems are handled by scalar methods; no vector diagrams are needed, nor are vector solutions appropriate.

(b) When the collision is elastic, both the conservation principles--momentum and kinetic energy--are fully applicable.

$$\text{(momentum)}\quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\text{(energy)}\quad m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

(Note: 1/2 factor has been eliminated by multiplying entire equation by 2.)

(c) Before starting the problem, note whether one or more of the velocities (u's or v's) are zero. If so, the entire term containing the zero factor should be dropped from the equation while it is still in its literal form.

(d) Note also whether you are dealing with equal masses. If so, drop the subscripts which differentiate the masses and combine terms where possible to obtain the simplest possible equations.

(e) Solve the equations simultaneously while still in the literal form and obtain an expression in which the unknown is alone on the left side of the statement.

(f) Substitute the given data and obtain the numerical answer if one is asked for.

next page

continued

INELASTIC COLLISIONS - ONE DIMENSION:

(g) See statement (a) above.

(h) Conservation of kinetic energy does *not* apply so that the energy equation in item (b) above cannot be used.

(i) Momentum *is* conserved, however. The final momentum of the system is equal to the initial momentum if no resultant, external forces are present. That is

$$p_f = p_i = m_1u_1 + m_2u_2$$

(j) Since momentum is conserved, the kinetic energy of the system *after* the collision may be determined from:

$$K_f = \frac{p_f^2}{2(m_1 + m_2)}$$

(k) In a situation where coupling of the masses occurs, the final mass is the sum of the two original masses and the final velocity is common for both masses.

PROBLEMS

1. In a one-dimensional elastic collision between two objects, mass m_2 is initially at rest. If u_1 = 1 km/sec, and $m_1 = 2m_2$, what is the final velocity of m_1?

2. Two masses, m_1 = 1 kg and m_2 = 2 kg, undergo a perfectly elastic collision in one dimension. Mass m_2 was initially at rest, and the velocity of m_1 before the collision was u_1 = 1 m/sec. Calculate the velocity v_2 of m_2 after the collision.

A. 2/3 m/sec
B. 1 m/sec
C. 3/2 m/sec
D. -1/3 m/sec

3. Two masses, m_1 and m_2, collide perfectly elastically. Mass m_2 is initially at rest. Select the answer which best describes the behavior of these two masses after collision when m_1 is negligibly small compared to m_2.

A. m_1 strikes m_2 and comes to rest, with m_2 remaining at rest.

B. m_1 strikes m_2 and rebounds in the opposite direction with the same speed.

C. m_1 strikes m_2 and comes to rest while m_2 moves forward slowly.

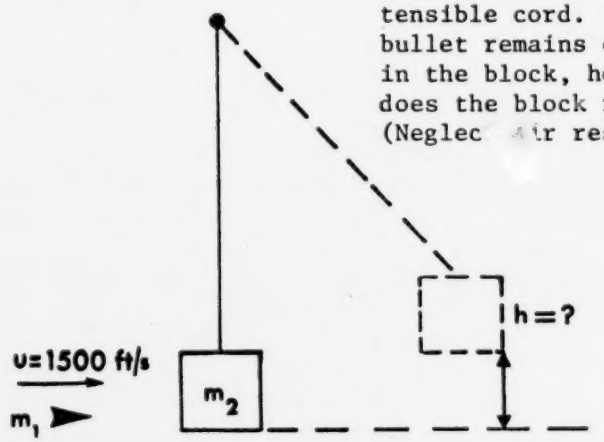
D. m_1 strikes m_2 and rebounds in the opposite direction with half its initial speed.

4. A fishnet buoy is hit by a sailboat traveling with a speed of 1 knot. Assuming that the collision is elastic and that the sailboat is much more massive than the buoy, what are the approximate velocities of boat and buoy after the impact?

A. sailboat, 1 knot forward; buoy, 1 knot forward
B. sailboat, 1 knot forward; buoy, 2 knots forward
C. sailboat, zero velocity; buoy, 2 knots forward
D. sailboat, 1 knot backward; buoy, stationary

5. A mass m_1 traveling in the positive x-direction collides head on and perfectly elastically with a stationary mass m_2. After the collision m_1 is moving toward the negative and m_2 toward the positive x-direction, both masses having the same speed. What is the value of the ratio m_2/m_1?

6. A 1/2-ounce bullet traveling horizontally with a muzzle speed of 1500 ft/sec strikes a 5-pound block suspended from a fixed point with 5-foot, massless, inextensible cord. If the bullet remains embedded in the block, how high does the block rise? (Neglect air resistance.)



A. 5.00 ft
B. 2.18 ft
C. 1.36 ft
D. 1.00 ft

7. A very massive body m_1 traveling with a velocity of 100 km/hr northward undergoes a perfectly inelastic collision with a body of much smaller mass m_2 ($m_2 << m_1$) moving at 100 km/hr southward. The respective velocities of the two bodies after the collision are approximately

A. 100 km/hr northward; 300 km/hr northward
B. zero; 200 km/hr northward
C. zero; zero
D. 100 km/hr northward; 100 km/hr northward

8. A block of mass m sliding on a frictionless table collides completely inelastically with a stationary block of mass 2m. What fraction of the system's initial energy is "lost" in the collisions?

A. 100%
B. 50%
C. 33%
D. 67%

9. A 10-gm bullet traveling horizontally with a speed of 500 m/sec strikes a 3-kg wooden block resting on an icy surface. (Neglect the frictional force between the block and ice surface.) The bullet remains embedded in the block and the combined body strikes a spring with a spring constant k = 30 nt/m. Find the maximum compression of the spring.

INFORMATION PANEL — Collision Problems - Two Dimensions

OBJECTIVE

To develop a practical, logical pattern for solving collision problems in two dimensions by working out a series of typical problems.

Collisions in two dimensions may be either elastic or inelastic just as in the one-dimensional case. Once again, if the collision is elastic, you may use the principle of conservation of kinetic energy but if it is inelastic, this conservation principle is not applicable. For both elastic and inelastic collisions in two dimensions, however, the principle of conservation of momentum is consistently valid.

Momentum is a vector quantity. The usual approach to collisions in two dimensions is to apply conservation considerations to the retangular components of the momentum separately. That is, momentum must be conserved along the x-axis and momentum must also be conserved along the y-axis. In many instances, a given problem can be substantially simplified by a judicious choice of coordinate axes as in the example shown on the next page.

next page

continued

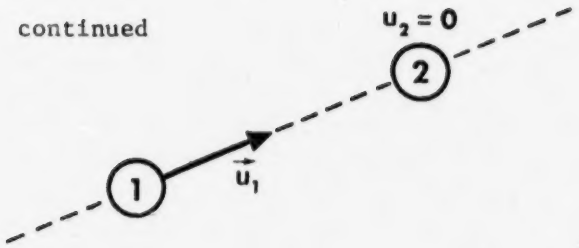


Figure 1

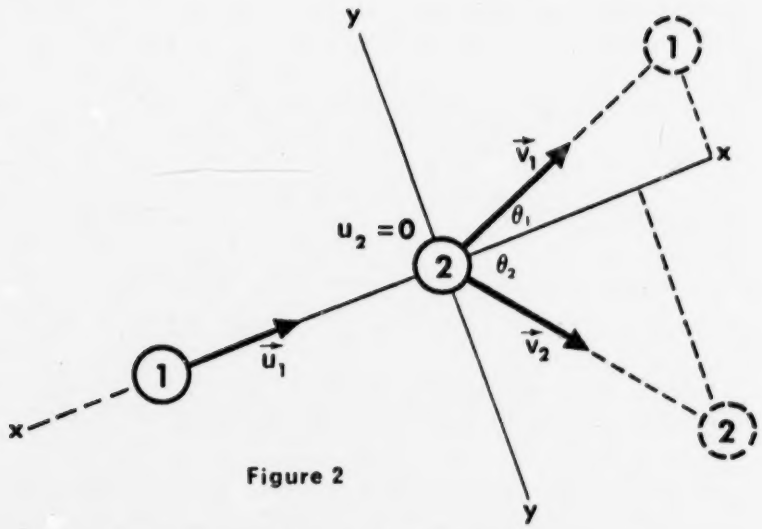


Figure 2

In Figure 1, two spheres are about to collide. Sphere 1 is moving along the line indicated while sphere 2 is at rest. Since the line of motion does not pass through the center of mass of sphere 2, the two bodies will move off after the collision as illustrated in Figure 2.

Note that the x-axis has been chosen so that it coincides with the original line of motion, thus making the y-component of u_1 equal to zero so that we have one less unknown to contend with.

The conditions for conservation of momentum along both axes may then be set up in a style similar to that shown on the next page.

If the collision was inelastic, you would come out with the two working equations shown. This tells you, of course, that the problem can be solved if it is limited to only two unknowns.

But if the collision was elastic, you could write an additional equation for the conservation of energy, therefore, permitting you to handle a problem in which there were three unknowns.

next page

continued

	p_x BEFORE COLLISION	p_x AFTER COLLISION
Body 1	m_1u_1	$m_1v_1 \cos\theta_1$
Body 2	0	$m_2v_2 \cos\theta_2$
TOTAL	m_1u_1 =	$m_1v_1 \cos\theta_1 + m_2v_2 \cos\theta_2$

Conservation of momentum along x-axis

	p_y BEFORE COLLISION	p_y AFTER COLLISION
Body 1	0	$m_1v_1 \sin\theta_1$
Body 2	0	$-m_2v_2 \sin\theta_2$ (Note negative sign)
TOTAL	0 =	$m_1v_1 \sin\theta_1 - m_2v_2 \sin\theta_2$

Conservation of momentum along y-axis

And, for an elastic collision only

$$\frac{1}{2} m_1u_1^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$$

Conservation of Kinetic Energy

10. A mass m_1 collides perfectly elastically with a stationary mass m_2. After the collision the two masses move at right angles to one another. What is the ratio m_2/m_1? (HINT: Choose $\vec{v}_1$ along the x-axis and $\vec{v}_2$ along the y-axis.)

11. A billiard ball moving at a speed of 8.7 m/sec collides with an identical stationary ball. After the collision, the first ball moves at an angle of 30° from its initial direction and the second ball moves at an angle of -30° from this direction. Find the speed of the first ball after collision.

12. A number of collision problems require an ability to solve the following algebraic problem. Given three equations,

$$v_1 \sin\theta_1 = v_2 \sin\theta_2 \qquad (1)$$

$$u_1 - v_1 \cos\theta_1 = v_2 \cos\theta_2 \qquad (2)$$

$$u_1^2 = v_1^2 + v_2^2$$

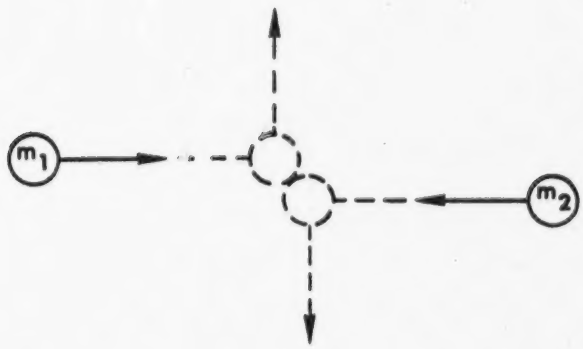
Where u_1 and θ_1 are known, and v_1, v_2, θ_2 are unknown.

Find v_1 in terms of known quantities.

13. A billiard ball of mass m moving at a speed of 5 m/sec collides elastically with an identical stationary ball. After the collision the first ball moves off at an angle of 30° from its initial direction. Find the speed of the second ball after collision.

14. A 1000-kg auto moving north at 60 km/hr collides perfectly inelastically with a 2000-kg truck moving east at 40 km/hr. How much mechanical energy is dissipated during the collision?

15. A body of mass m_1 moving due east with speed u_1 = 10 m/sec collides with another body of mass m_2 moving due west with a speed u_2 = 30 m/sec. If after the collision m_1 travels directly northward and m_2 travels directly southward, what is the ratio of the particles' masses, m_1/m_2? (This information will be used in the next question.)



A. 12
B. 9
C. 3
D. 1/3

16. If during the collision in the preceding question 30% of the total kinetic energy is dissipated as heat, what are the respective speeds of the two bodies after the collision?

A. 10 m/sec; 30 m/sec
B. 30 m/sec; 10 m/sec
C. 8.65 m/sec; 25.95 m/sec
D. 25.95 m/sec; 8.65 m/sec

17. A 1000-kg auto moving east at 50 km/hr collides perfectly inelastically with a similar automobile of same mass moving at 50 km/hr in a direction 60° north of east. Find the kinetic energy of the system after collision.

[a] CORRECT ANSWER: 3

We have a perfectly elastic collision, in which $u_2 = 0$ and $v_1 = -v_2$. Thus, from conservation of momentum,

$$m_1u_1 = m_1v_1 + m_2v_2 = (m_1 - m_2)\, v_1 \qquad (1)$$

and from conservation of energy

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2 = (m_1 + m_2)\, v_1^2 \qquad (2)$$

We square (1) and multiply (2) by m_1 to obtain

$$m_1^2u_1^2 = (m_1 - m_2)^2\, v_1^2 \qquad (3)$$

$$m_1^2u_1^2 = m_1(m_1 + m_2)\, v_1^2 \qquad (4)$$

Dividing (3) by (4) and expanding the square yields

$$m_1^2 + m_1m_2 = m_1^2 + m_2^2 - 2m_1m_2$$

or

$$3m_1m_2 = m_2^2$$

Finally

$$\frac{m_2}{m_1} = 3$$

TRUE OR FALSE: In this solution, the data enable us to write that $v_1 = -v_2 = 0$

[b] CORRECT ANSWER: B

Since the mass of the buoy is negligible compared to that of the sailboat, we know that the sailboat will continue with almost undiminished velocity and that the buoy will rebound with approximately twice the initial velocity of the sailboat.

[a] CORRECT ANSWER: 1/3 km/sec

Both momentum and mechanical energy are conserved. Thus,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{1}$$

and

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2 \tag{2}$$

Substituting $m_1 = 2m_2$ and $u_2 = 0$, we obtain

$$2m_2u_1 = 2m_2v_1 + m_2v_2 \tag{3}$$

and

$$2m_2u_1^2 = 2m_2v_1^2 + m_2v_2^2 \tag{4}$$

Or,

$$v_2 = 2(u_1 - v_1) \tag{5}$$

and

$$v_2^2 = 2(u_1^2 - v_1^2) = 2(u_1 - v_1)(u_1 + v_1) \tag{6}$$

Squaring (5) and dividing by (6) we find

$$1 = \frac{4(u_1 - v_1)^2}{2(u_1 - v_1)(u_1 + v_1)} \quad \frac{2(u_1 - v_1)}{u_1 + v_1}$$

or

$$u_1 + v_1 = 2u_1 - 2v_1$$

Finally,

$$v_1 = \frac{1}{3}\,u_1 = \frac{1}{3} \times (1 \text{ km/sec}) = \frac{1}{3} \text{ km/sec}$$

TRUE OR FALSE? Equation (4) is derived from the mathematical statement of the principle of conservation of kinetic energy.

[a] CORRECT ANSWER: C

The total momentum of the system before the collision (both in the x- and y-direction) was zero. Hence, the momentum after the collision must also be zero. This gives

$$m_1 v_1 - m_2 v_2 = 0$$

or, using the results obtained in the solution of the previous problem ($m_1 = 3m_2$)

So,

$$v_2 = 3v_1 \qquad (1)$$

The final energy is seventy percent of the initial energy,

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = 0.7 \ \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

Again using the fact that $m_1 = 3m_2$ we obtain

$$3v_1^2 + v_2^2 = 2.1u_1^2 + 0.7u_2^2$$

Substituting the values of u_1 and u_2 along with equation (1) we find

$$12v_1^2 = 210 + 630 = 840$$

or

$$v_1 = \sqrt{70} = 8.65 \text{ m/sec}$$

and

$$v_2 = 3v_1 = 25.95 \text{ m/sec}$$

[a] CORRECT ANSWER: C

After the collision the x-component of the momentum of the system is zero. Conservation of momentum requires that the x-component of the system's momentum before the collision also be zero. Taking eastward as the positive direction, we have

$$m_1u_1 - m_2u_2 = 0$$

or

$$m_1u_1 = m_2u_2$$

Thus,

$$\frac{m_1}{m_2} = \frac{u_2}{u_1} = \frac{30 \text{ m/sec}}{10 \text{ m/sec}} = 3$$

[b] CORRECT ANSWER: A

With $u_2 = 0$, the energy and momentum equations (dropping the 1/2 from the energy equation) become

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2 \qquad (1)$$

$$m_1u_1 = m_1v_1 + m_2v_2 \qquad (2)$$

Eliminating v_1 from these two equations (see the first equation on page 184 of HR with $u_2 = 0$) we find

$$v_2 = \left[\frac{2m_1}{m_1 + m_2}\right]u_1 \qquad (3)$$

Substituting the given data into (3) we get

$$v_2 = \left[\frac{(2) \times (1 \text{ kg})}{(2 \text{ kg}) + (1 \text{ kg})}\right] \times (1 \text{ m/sec}) = \frac{2}{3} \text{ m/sec}$$

[a] CORRECT ANSWER: D

In a completely (or perfectly) inelastic collision the two colliding bodies stick together so their velocities after the collision are equal; i.e., $v_1 = v_2$. Using this, along with the given information $m_2 = 2m_1 \equiv 2m$, and $u_2 = 0$, in the conservation of momentum equation, we find

$$mu_1 = mv_1 + 2mv_1 = 3mv_1$$

or

$$v_1 = v_2 = \frac{u_1}{3} \qquad (1)$$

The energy "lost" during the collision is $K_i - K_f$. Or expressed in percent of the initial energy,

$$\text{percent "loss"} = 100 \times \frac{K_i - K_f}{K_i} \qquad (2)$$

Now,

$$K_i - K_f = \frac{1}{2} mu_1^2 - \left[\frac{1}{2} mv_1^2 + \frac{1}{2} (2m)v_1^2\right] = \frac{1}{2} m \left[u_1^2 - 3v_1^2\right]$$

$$= \frac{1}{2} m \left[u_1^2 - 3(u_1/3)^2\right] = \frac{1}{2} m \left[\frac{2}{3} u_1^2\right]$$

So,

$$\text{percent "loss"} = 100 \times \frac{\frac{1}{2} m \left[\frac{2}{3} u_1^2\right]}{\frac{1}{2} mu_1^2} = 100 \times \frac{2}{3} = 67\%$$

[a] CORRECT ANSWER: C

During an inelastic collision we cannot apply the principle of conservation of mechanical energy. However, immediately after the collision, dissipative forces are no longer in effect and we can write

$$E = E_f \qquad (1)$$

where E is the total energy for the system immediately after the bullet is embedded, and E_f is the total energy of the block-bullet system when it has reached its maximum height, h. We take the "ground" level to be the original position of the block, so Eq. (1) becomes

$$(1/2)(m_1 + m_2)v^2 = (m_1 + m_2)\, gh$$

or

$$h = v^2/(2g) \qquad (2)$$

The only unknown on the right side of this expression is v, the speed of the bullet and block after embedding. To find v, we can apply the principle of conservation of momentum. Notice that although an outside force due to gravity exists, this force is not directed along the horizontal. Thus, momentum is conserved on the horizontal direction;

$$m_1 u_1 = (m_1 + m_2)\, v$$

or

$$v = m_1 u_1/(m_1 + m_1)$$

Finally, substitute this into Eq. (2),

$$h = \frac{m_1^2 u_1^2}{2(m_1 + m_2)^2\, g} = 1.36 \text{ ft}$$

TRUE OR FALSE? The method used for solving this problem here depends on the use of the conservation of momentum principle for determining the quantity v.

[a] CORRECT ANSWER: 1.33×10^5 j

In this, like in any other collision, we utilize conservation of momentum. Therefore, the final momentum of the system, $\vec{p}_f$, will be equal to its initial momentum $\vec{p}_i$, which is the vectorial sum of the two individual momenta. The factor converting km/hr to m/sec is

$$1 \frac{km}{hr} = 1 \frac{km}{hr} \times \frac{10^3}{1} \frac{m}{km} \times \frac{1}{3.6 \times 10^3} \frac{hr}{sec} = \frac{1}{3.6} \frac{m}{sec}$$

Assigning subscripts 1 and 2 to the car and truck, respectively, we find

$$p_1 = m_1 v_1 = (10^3)\left[60 \times \frac{1}{3.6}\right] = \frac{6}{3.6} \times 10^4 \frac{kg\text{-}m}{sec}$$

and

$$p_2 = m_2 v_2 = (2 \times 10^3)\left[40 \times \frac{1}{3.6}\right] = \frac{8}{3.6} \times 10^4 \frac{kg\text{-}m}{sec}$$

The magnitudes of the initial and final momenta are (see diagram)

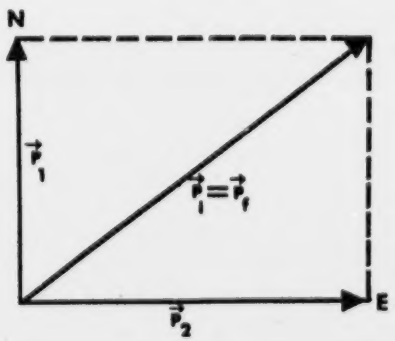
$$p_i = p_f = \sqrt{{p_1}^2 + {p_2}^2} = \sqrt{\frac{36 \times 10^8 + 64 \times 10^8}{(3.6)^2}} = \frac{10^5}{3.6} \frac{kg\text{-}m}{sec}$$

Using the relation $K = p^2/2m$, and the fact that after the collision the two vehicles are stuck together, we obtain

$$K_i - K_f = \left(\frac{{p_1}^2}{2m_1} + \frac{{p_2}^2}{2m_2}\right) - \left(\frac{{p_f}^2}{2(m_1 + m_2)}\right)$$

$$= \left(\frac{1}{3.6}\right)^2 \left(\frac{36 \times 10^8}{2000} + \frac{64 \times 10^8}{4000} - \frac{100 \times 10^8}{6000}\right)$$

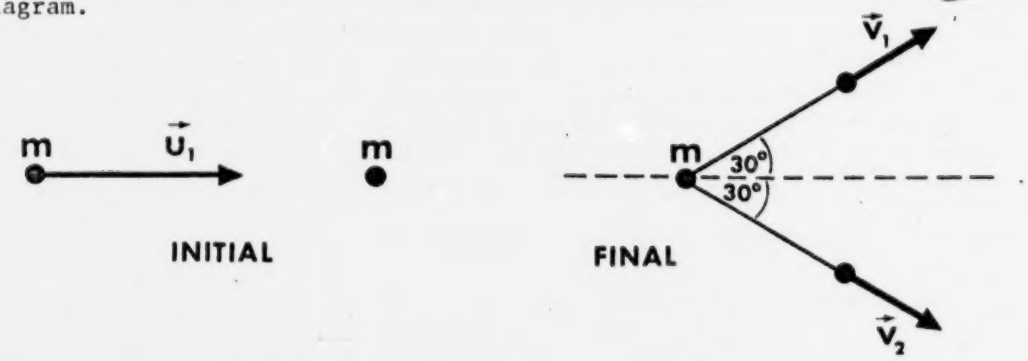
$$= 1.33 \times 10^5 \text{ j}$$



TRUE OR FALSE? The final velocity of the auto is the same as that of the truck in this problem.

[a] CORRECT ANSWER: 5.0 m/sec

Associate subscripts 1 and 2 with particles one and two as shown in the diagram.



Applying conservation of momentum for the x-component,

$$mu_1 = mv_1 \cos 30° + mv_2 \cos 30° \qquad (1)$$

and for the y-component

$$0 = mv_1 \sin 30° - mv_2 \sin 30° \qquad (2)$$

These two equations have two unknowns, v_1 and v_2. From Eq. (2),

$$v_1 = v_2$$

Substituting this into Eq. (1) gives

$$u_1 = 2v_1 \cos 30°$$

$$v_1 = \frac{u_1}{2 \cos 30°} = 5.0 \text{ m/sec}$$

[a] CORRECT ANSWER: D

The qualitative picture of this collision is so simple that you may have been able to select the right answer without any calculations at all. If m_2 is indeed *very, very* small compared to m_1, its effect upon the initial momentum of the large mass after collision would be insignificant since the two velocities are the same in magnitude. Hence, m_1 continues unaffected at a speed of 100 km/hr northward, carrying m_2 with it. If you are interested in seeing how the same answer is obtained mathematically, you might study the solution below.

For any collision momentum is conserved, so

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad (1)$$

Taking the northward as our positive direction, u_1 = 100 km/hr and u_2 = -100 km/hr. Therefore, $u_2 = -u_1$. Also, from the fact that the collision is completely inelastic, we know that $v_1 = v_2$. Using this information in (1) we obtain

$$(m_1 - m_2)\, u_1 = (m_1 + m_2)\, v_1$$

or

$$v_2 = v_1 = \frac{m_1 - m_2}{m_1 + m_2}\, u_1 = \frac{1 - m_2/m_1}{1 + m_2/m_1}\, u_1 \qquad (2)$$

In the limit $m_2/m_1 \to 0$ equation (2) gives

$$v_2 = v_1 = \frac{1 - 0}{1 + 0}\, u_1 = u_1 = 100 \text{ km/hr (northward)}$$

[a] CORRECT ANSWER: B

This can be easily demonstrated using the equation

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 \qquad \text{(HR page 184)}$$

As m_1 becomes negligible compared to m_2, $m_1/m_2 \rightarrow 0$ and

$$\frac{v_1}{u_1} = \frac{m_1 - m_2}{m_1 + m_2} = \frac{m_1/m_2 - 1}{m_1/m_2 + 1} \longrightarrow \frac{-1}{1} = -1$$

so

$$v_1 \rightarrow -u_1$$

If we follow a similar procedure for v_2 using

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right) u_1$$

we can see that, as m_2 becomes large compared to m_1, the entire term in the brackets becomes negligible. Thus, $v_2 \approx 0$. There are several good examples of the above discussion. Of course, in practice, no collision is perfectly elastic (except some of those between atoms), but many are close enough to being perfectly elastic that for all practical purposes they may be considered so.

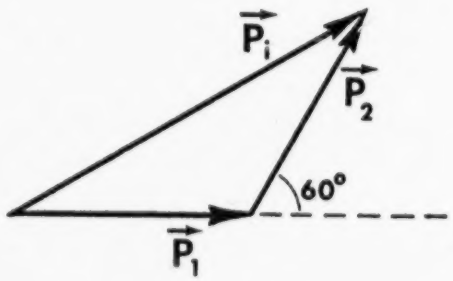
For our first example, think of a billiard ball thrown against the bumper of a car that is free to roll. Here the ratio of the mass of the car to the mass of the ball is huge. From our experience, we can see that the results predicted by the two equations above will be borne out to a good degree of approximation. The ball will bounce off the car with about the same speed it had before the collision. The car will not appear to move.

As our second example, we may think of a lively basketball thrown against the hull of a very large ship. Here, the ratio of the masses might be as much as 10^8:1. Again, the results of this approximately elastic collision are very accurately predicted by the above equations.

[a] CORRECT ANSWER: 1.4×10^5 j

Assigning subscripts 1 and 2 to the car moving east and the car moving 60° north of east, respectively, the magnitude of the total momentum before collision p_i is given by

$$p_i = \sqrt{p_1^2 + p_2^2 + 2p_1p_2 \cos\theta}$$



where θ is the angle between $p_1 + p_2$.

Conservation of momentum yields

$$p_i = p_f$$

and the required kinetic energy is given by

$$K = \frac{p_f^2}{2(m_1 + m_2)} = \frac{p_i^2}{2(m_1 + m_2)} = \frac{p_1^2 + p_2^2 + 2p_1p_2 \cos\theta}{2(m_1 + m_2)}$$

or

$$K = \frac{3p_1^2}{2(m_1 + m_2)}$$

since $p_1 = p_2$ and $\cos 60° = \frac{1}{2}$

$$K = \frac{3}{4} m_1 v_1^2 \qquad (1)$$

since $p_1 = m_1 v_1$ and $m_1 = m_2$

$$K = \frac{3}{4} (10^3 \text{ kg}) \left(50 \times 10^3 \frac{\text{m}}{\text{hr}} \times \frac{1}{3.6 \times 10} \frac{\text{hr}}{\text{sec}}\right)^2$$

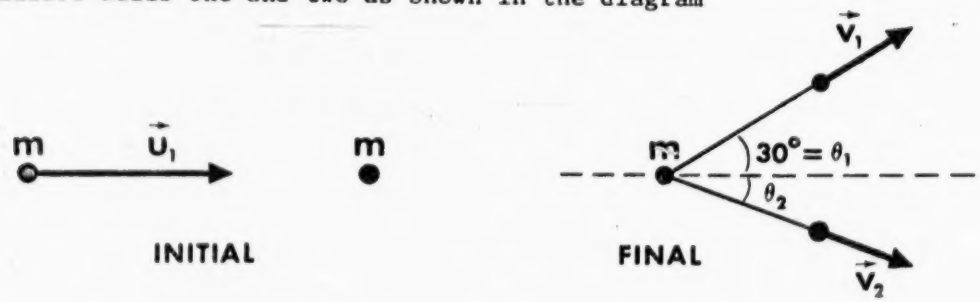
$$= 1.44 \times 10^5 \text{ nt-m or j}$$

Note that the kinetic energy is not conserved in a perfectly inelastic collision. This is shown in equation (1).

TRUE OR FALSE? The angle between the momentum of the eastward moving car and the initial momentum of the whole system is 60°.

[a] CORRECT ANSWER: 2.5 m/sec

Let subscripts 1 and 2 describe the velocities associated with the billiard balls one and two as shown in the diagram



Applying the conservation of momentum for the x-component of motion we have

$$mu_1 = mv_1 \cos\theta_1 + mv_2 \cos\theta_2$$

$$u_1 - v_1 \cos\theta_1 = v_2 \cos\theta_2 \qquad (1)$$

and for the y-component

$$0 = mv_1 \sin\theta_1 - mv_2 \sin\theta_2$$

$$v_1 \sin\theta_1 = v_2 \sin\theta_2 \qquad (2)$$

Since the collision is elastic, the conservation of kinetic energy applies and we have

$$\frac{1}{2} mu_1^2 = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$$

or

$$u_1^2 = v_1^2 + v_2^2 \qquad (3)$$

next page

continued

As we are asked to obtain v_2, we may eliminate θ_2 by the following operations:

Squaring equations (1) and (2) and adding, we obtain

$$v_2^2(\sin^2\theta_2 + \cos^2\theta_2) = v_1^2(\sin^2\theta_1 + \cos^2\theta_1) + u_1^2 - 2u_1v_1\cos\theta_1$$

or

$$v_2^2 = v_1^2 + u_1^2 - 2u_1v_1\cos\theta_1 \qquad (4)$$

where the identity $\sin^2\theta + \cos^2\theta = 1$ has been used.

Substituting the value of u_1^2 from equation (3) in equation (4) we obtain

$$v_2^2 = v_1^2 + v_1^2 + v_2^2 - 2u_1v_1\cos\theta_1$$

or

$$v_1 = u_1\cos\theta_1$$

Substituting this value for v_1 in equation (3) and solving for v_2, we obtain

$$v_2 = \sqrt{u_1^2(1 - \cos^2\theta_1)}$$

$$= u_1\sin\theta_1 = 5 \text{ m/sec} \times \frac{1}{2} = 2.5 \text{ m/sec}$$

TRUE OR FALSE? The conditions shown in this solution indicate that the line of motion passes through the centers of mass of both billiard balls.

[a] CORRECT ANSWER: $v_1 = u_1 \cos\theta_1$

Eliminate θ_2 by the following operations:

Square both sides of equations (1) and (2),

$$v_1^2 \sin^2\theta_1 = v_2^2 \sin^2\theta_2$$

$$u_1^2 - 2u_1v_1 \cos\theta_1 + v_1^2 \cos^2\theta_1 = v_2^2 \cos^2\theta_2$$

and add the results,

$$u_1^2 - 2u_1v_1 \cos\theta + v_1^2(\cos^2\theta_1 + \sin^2\theta_1) = v_2^2(\cos^2\theta_2 + \sin^2\theta_2)$$

Now use the trigonometric identity

$$\cos^2\theta + \sin^2\theta = 1$$

to obtain

$$u_1^2 - 2u_1v_1 \cos\theta_1 + v_1^2 = v_2^2$$

Finally, substitute u_1^2 from equation (3) into this to obtain

$$v_1 = u_1 \cos\theta_1$$

[b] CORRECT ANSWER: 53 cm

The collision between the bullet and the block is perfectly inelastic. In such a collision, the momentum is conserved, yielding

$$p_f = p_i = m_1u_1 \qquad (1)$$

The kinetic energy of the combined system immediately after collision is

$$k_f = \frac{p_f^2}{2(m_1 + m_2)} = \frac{(m_1u_1)^2}{2(m_1 + m_2)} \qquad (2)$$

next page

continued

When the block-bullet system causes maximum compression, all of its kinetic energy has been converted into the potential energy of the spring. Thus,

$$\frac{1}{2}kx^2 = \frac{(m_1u_1)^2}{2(m_1 + m_2)}$$

and

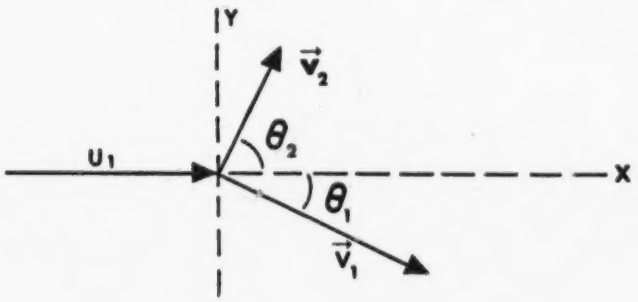
$$x = \frac{m_1u_1}{\sqrt{k(m_1 + m_2)}} \qquad (3)$$

$$= 53 \text{ cm}$$

TRUE OR FALSE? At the instant when the spring reaches maximum compression, the kinetic energy of the block-bullet system is zero.

[a] CORRECT ANSWER: 1

In order to demonstrate the usefulness of choosing the appropriate coordinate axes, we shall present two ways of solving this problem.



In the first method, we follow the usual (up to now) procedure of choosing the velocity of the oncoming particle, $\vec{u}_1$, along the positive x-axis. The initial momentum then is

$$\vec{P}_i = m_1u_1\vec{I} \qquad (1)$$

The final momentum is

$$\vec{P}_f = m_1\vec{v}_1 + m_2\vec{v}_2 =$$

$$(m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2)\,\vec{I} + (-m_1v_1\sin\theta_1 + m_2v_2\sin\theta_2)\,\vec{J} \qquad (2)$$

next page

continued

Conservation of momentum and energy yields

$$m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 = m_1 u_1 \quad (3)$$

$$m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 = 0 \quad (4)$$

$$m_1 v_1^2 + m_2 v_2^2 = m_1 u_1^2 \quad (5)$$

Squaring and adding (3) and (4), and using the fact that sin + cos = 1, we obtain

$$m_1^2 v_1^2 + m_2^2 v_2^2 + 2m_1 m_2 v_1 v_2 (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) = m_1^2 u_1^2 \quad (6)$$

When the identity $(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) = \cos(\theta_1 + \theta_2)$ is used, along with $\theta_1 + \theta_2 = 90^\circ$ or $\cos(\theta_1 + \theta_2) = 0$, (6) becomes

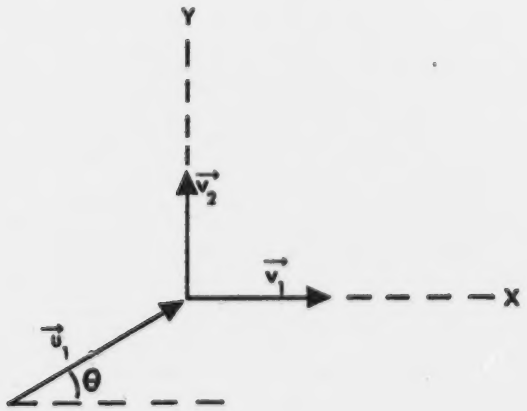
$$m_1^2 v_1^2 + m_2^2 v_2^2 = m_1^2 u_1^2 \quad (7)$$

Finally, multiplying (5) by m_1 and subtracting it from (7), we obtain

$$m_2^2 v_2^2 - m_1 m_2 v_2^2 = 0$$

or

$$m_1 = m_2$$



We now employ the coordinate system suggested in the hint. Conservation of momentum and energy results in

$$m_1 u_1 \cos\theta = m_1 u_1 \quad (8)$$

$$m_1 u_1 \sin\theta = m_2 v_2 \quad (9)$$

$$m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2 \quad (10)$$

next page

continued

Squaring and adding (8) and (9) we obtain

$$m_1^2 u_1^2 = m_1^2 v_1^2 + m_2^2 v_2^2 \qquad (11)$$

But this is the same as equation (7), with (10) being the same as (5). We see, therefore, that we arrive at the same results, saving much labor, simply by choosing the coordinate axes judiciously.

TRUE OR FALSE? In the second solution, the angle θ is a known quantity from the very beginning.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL The Universal Constant of Gravitation

OBJECTIVE

To recognize that the constant of gravitation G is universal in nature and must be determined experimentally.

From Newton's law of gravitation we know that in a gravitating system consisting of two point masses m_1 and m_2 separated by a distance r, the force on either particle varies directly as the product of the two masses ($m_1 m_2$) and inversely as r^2, or

$$F \propto \frac{m_1 m_2}{r^2}$$

This is a proportion, *not an equation*, but it may be readily written in equation form by introducing a constant of proportionality k so that

$$F = k \frac{m_1 m_2}{r^2}$$

Conventionally, G is used instead of k to identify the gravitational nature of the force, or

$$F = G \frac{m_1 m_2}{r^2}$$

Thus, the physical significance of G is that it is a constant of proportionality independent of the characteristics of the Earth, the moon, or other planets. By its very nature, then, it is universal in application just as long as the proportion describing the force of gravitation remains universally valid--and we have yet no evidence to the contrary.

In the MKS system of units G is neither unity nor dimensionless. If F is to come out in newtons, then the unit for G must be $nt\text{-}m^2/kg^2$ since

$$F = G \frac{m_1 m_2}{r^2}$$

then

$$[G] = \left[\frac{Fr^2}{m_1 m_2}\right] = \frac{nt\text{-}m^2}{kg^2}$$

next page

continued

The numerical value of G must be determined experimentally in a laboratory equipped to measure F for known masses separated by a known distance. The accepted value obtained in this way is $G = 6.673 \times 10^{-11}$ nt-m^2/kg^2. Once measured in our terrestrial laboratory, this value may be applied to gravitational configurationa anywhere in the universe.

In this section you will be asked to

(a) calculate the gravitational force between an electron and a proton in a simple atom;

(b) display understanding of the universal nature of G;

(c) determine the magnitude of the force of gravitation between the moon and the Earth.

PROBLEMS

1. In the Bohr picture of the hydrogen atom, the electron revolves about the proton in a circular orbit of radius 5.3×10^{-11} m and period 1.5×10^{-16} sec. The mass of the electron is $m_e = 9.1 \times 10^{-31}$ kg, and that of the proton is $m_p = 1.7 \times 10^{-27}$ kg. Calculate the gravitational force of the proton on the electron.

2. Cavendish made the first accurate measurement of G some 130 years after Newton proposed his law of universal gravitation. Let's assume that a Martian physicist is also able to perform the Cavendish experiment. Which of the following values of G would he obtain on Mars?

A. 6.7×10^{-11} nt-m^2/kg^2

B. 2.6×10^{-11} nt-m^2/kg^2

C. 1.9×10^{-11} nt-m^2/kg^2

D. 24×10^{-11} nt-m^2/kg^2

3. What is the magnitude of the mutual gravitational force between the moon and the Earth? Masses of the Earth and the moon are $M_e = 6 \times 10^{24}$ kg and $M_m = 7.3 \times 10^{22}$ kg respectively, and the distance between the centers of the Earth and moon is $r_{em} = 3.8 \times 10^8$ m.

INFORMATION PANEL — Acceleration Due to Gravity

OBJECTIVE

To solve problems in which gravitational acceleration is a key factor.

Unlike the constant of universal gravitation G, the acceleration of a given mass as it falls toward a planet or any other massive celestial object may vary over a wide range of values. The factors which govern the magnitude of gravitational acceleration may be summarized as follows:

(a) *Distance between centers of mass.* In elementary problems, all planets and accelerating masses are considered as perfect spheres with symmetrical mass distribution so that calculations may be based on point mass considerations. It is easily shown that

$$\frac{a}{g} = \frac{R^2}{r^2}$$

where

a = gravitational acceleration of a given mass at any altitude
g = gravitational acceleration of a given mass near the surface of the planet
R = radius of the planet
r = distance between centers of mass (planet and falling body) at the given altitude

If altitude is symbolized by h, then we can write

$$\frac{a}{g} = \frac{R^2}{(R + h)^2}$$

next page

continued

(b) *Mass of the planet.* Using the law of gravitation:

$$F_g = G\frac{mM}{R^2}$$ where m = mass of falling body, M = mass of planet, R = distance between centers

and recognizing that $F_g = w = mg$ for the falling body, then

$$mg = G\frac{mM}{R^2} \quad \text{and so} \quad g = \frac{GM}{R^2}$$

This shows that g is directly proportional to the mass of the planet toward which the moving body is falling.

In the section that follows, you will be expected to determine

(a) the orbital radius of a satellite, given the mass of the mother planet, the value of g for that planet, and the period of revolution of the satellite;

(b) the weight of a given mass at a specified altitude above the Earth;

(c) the value of g for the moon, given its radius and mass;

(d) the mass of a planet other than the Earth, given all the required data.

4. If a Martian were working on the law of universal gravitation by considering that Mars acted as a particle attracting a Martian apple and a Martian moon according to the same law, he could calculate the distance of that moon from Mars on the assumption that the gravitational attraction falls off as the inverse square of the distance from the center of Mars. From measurements made on Mars he obtains $g = 3.8$ m/sec^2 for the acceleration of a falling apple, $R = 3.4 \times 10^6$ m for the radius of Mars and $T = 2.76 \times 10^4$ sec for the period of Phobos (the larger of Mars' two moons). What value would he obtain for the radius of Phobos' orbit about Mars? (assumed circular)

A. 1,580 km
B. 9,500 km
C. 16,000 km
D. 17,500 km

5. True or false? As the distance of a body from the Earth increases, g *increases.*

6. How much does a 100-kg mass weigh at an altitude of 6400 km above the Earth's surface? (Take the radius of the Earth to be 6400 km.)

7. Assume the moon to be a sphere of radius 1,700 km and a uniformly distributed mass of 7.35×10^{22} kg. What is the acceleration due to gravity (g) at the surface of the moon?

8. Having measured G, Cavendish was able to "weigh" the Earth, i.e., calculate its mass. What value would a Martian physicist obtain for the mass of Mars given the following data?

acceleration due to gravity on the surface of Mars	$g = 3.8$ m/sec^2
radius of Mars	$R = 3400$ km
period of revolution of Phobos	$T = 2.76 \times 10^4$ sec
distance of Phobos from Mars	$r = 9500$ km
universal gravitational constant	$G = 6.7 \times 10^{-11}$ nt-m^2/kg^2

A. 1.9×10^{17} kg

B. 7.22×10^{17} kg

C. 6.55×10^{23} kg

D. 5.1×10^{24} kg

9. The orbital radius of a satellite of mass m around the Earth is twice the radius of the Earth. Which of the following correctly expresses the velocity of the satellite in terms of R_e (radius of the Earth) and g, the acceleration due to gravity on the surface of the Earth.

A. $v = \frac{gR_e}{2}$

B. $v = \frac{gM_e}{2R_e}$

C. $v = \sqrt{\frac{gR_e}{2}}$

D. $v = gR_e$

10. Inertial and gravitational masses are conceptually distinct, although experimentally have the same value. We use one symbol, m, to denote both kinds of masses. In which of the following euqations does m stand for gravitational mass?

1. $\vec{F} = m\vec{a}$
2. $\vec{p} = m\vec{v}$
3. $K = \frac{1}{2} mv^2$
4. $F = \frac{mv^2}{r}$
5. $F = \frac{GMm}{r^2}$
6. $g = \frac{Gm}{R_e^2}$
7. $U = mgh$
8. $T = 2\pi\sqrt{m/k}$

A. All of them
B. Numbers 3, 5, and 7
C. Numbers 5, 6, and 7
D. None of them

INFORMATION PANEL **Weight on a Rotating Planet**

OBJECTIVE

To determine the effect of rotation of a planet on the weight of a body located on its surface.

It has been shown that the weight of a body on the surface of a planet may be expressed as

$$W = mg = G\frac{mM_p}{R_p^2} \qquad (1)$$

where m = mass of body; M_p = mass of planet; R_p = radius of planet. In setting up this equation, no attention was given to the possible effect of planetary rotation on its axis upon the apparent weight of the body.

An object located directly over either pole is not subjected to centripetal acceleration because it has no tangential velocity. That is

$$a_R = \frac{v^2}{r} = \frac{0}{r} = 0$$

so that its radial acceleration is zero. For this body, equation (1) gives the apparent weight. Its *true* weight (F_g of planet on body) and *apparent* weight (reading of a spring balance on which it hangs) are the same.

As the object is moved along the surface, away from the pole and toward the equator, it experiences an increasing centripetal acceleration which reaches maximum at the equator where its tangential velocity is greatest. This means that it must be acted upon by an *unbalanced* radial force directed toward the center of the earth.

The second law tells us that this net radial force is equal to ma_R. For an object suspended on a spring balance, the net force is the difference between the force of gravity F_g and the upward pull of the stretched spring (apparent weight w_a). Thus

$$F_g - w_a = ma_R \qquad (2)$$

or

$$w_a = F_g - ma_R \qquad (3)$$

next page

continued

Equation (3) shows clearly that *the apparent weight w_a as read on the balance is less than its true weight F_g by an amount equal to the magnitude of the centripetal force ma_R acting on the body.*

The problems that follow require that you

(a) find the difference in apparent weight of a man at one pole of the earth as contrasted with his apparent weight at the equator;

(b) recognize the difference between weighing a body with a spring balance and weighing the same body in the same circumstances with a platform or lever balance.

11. For a perfectly spherical Earth of radius 6.37×10^6 m with its axis through both poles, how much more (or less) would a 70-kg man weigh at either pole than he would on the equator? (Assume the weighing to be done with a "massless" spring balance.)

A. he would weigh the same
B. 3.62 nt more
C. 2.36 nt more
D. 2.36 nt less

12. Suppose the 70-kg mass in question 11 uses a lever balance (a balance in which counterweights are used to measure an unknown weight) to weigh himself. What would then be the difference in the reading of the balance at the poles and at a point on the equator?

A. 2.36 nt
B. 1.18 nt
C. 4.72 nt
D. zero

13. It is planned to place a satellite in circular orbit always directly above a certain point on the Earth's equator. (Take the mass of the earth to be 6×10^{24} kg.) This plan will be

A. successful at any altitude
B. unsuccessful at any altitude
C. successful if the satellite's altitude is 35,000 km
D. successful if the satellite's altitude is 42,000 km

14. Assuming a perfectly spherical planet of radius R, and mass M, what will a body of mass m weighing W at one of the poles of the planet weigh at its equator? The angular frequency of rotation of the planet is ω and it rotates about an axis through both the poles.

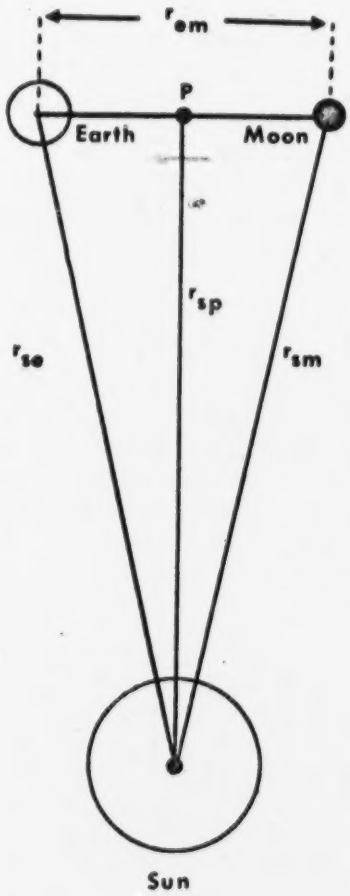
A. $W - g$

B. $W - mg$

C. $W - \omega mg$

D. $W\left[1 - \frac{\omega^2 R}{g}\right]$

15.



Referring to the diagram, point P is midway along the line from the Earth to the moon. This line is normal to the radius vector from the sun to P, so the center of the Earth and the moon are equidistant from the sun. Neglecting the effects of other members of the solar system, compute the gravitational field strength at P.

A. 1.25×10^{-2} nt/kg; at an angle of 62° to the left of the line going from P to the sun

B. 1.25×10^{-2} nt/kg; at an angle of 62° below the line going from P to the moon

C. 5.9×10^{-3} nt/kg; directly toward the sun

D. 5.9×10^{-3} nt/kg; directly away from the sun

16. The mass of Saturn is 5.7×10^{26} kg. If the planet were a sphere of radius 58,300 km, and had a spherically symmetric mass distribution, what would be the magnitude of the gravitational field at the surface of Saturn?

17. From the following astronomical data, find the point on a line joining the centers of the Earth and the moon at which the gravitational field strength of the Earth-moon system is zero. Let the distance from the center of the Earth to the center of the moon be 3.8×10^5 km, the mass of the Earth be 6×10^{24} kg, and the mass of the moon be 7.3×10^{22} kg.

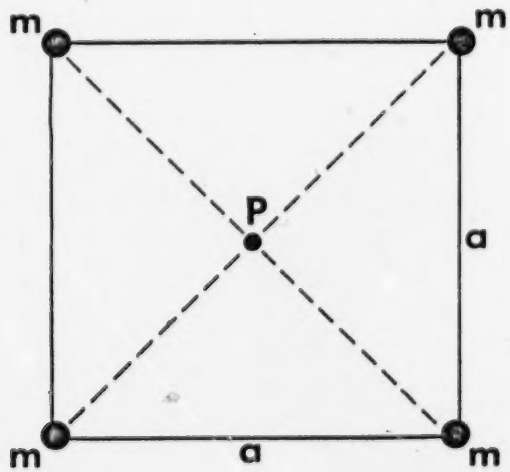
A. 3.4×10^8 m from the center of the Earth.

B. 3.4×10^8 m from the center of the moon.

C. 3.8×10^8 m from the center of the Earth.

D. 3.8×10^8 m from the center of the moon.

18. Four equal masses (each of mass m) are placed at the corners of a square with sides of length a. Neglecting the effect of all other masses in the universe, find the gravitational field at P, the geometric center of the square.



A. zero

B. unity

C. $G \frac{4M}{r^2}$

D. $-G \frac{4M}{r^2}$

INFORMATION PANEL

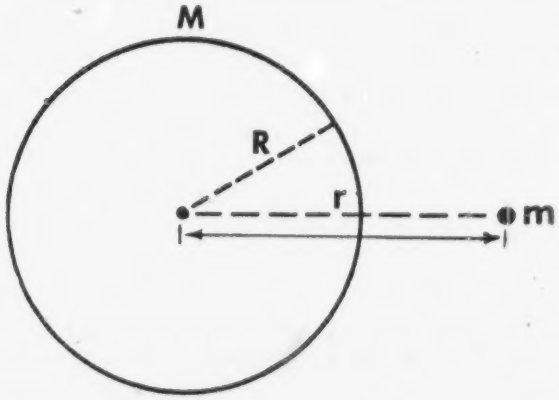
Gravitational Effects of Spherically Symmetric Mass Distributions

---

OBJECTIVE

To solve problems involving gravitational fields inside and outside hollow and solid spheres having symmetrical mass distributions.

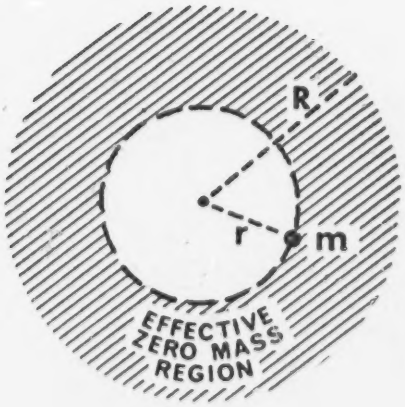
This Information Panel summarizes the salient concepts and relationships encountered in problems dealing with the gravitational effects associated with spherically symmetric mass distributions. Derivations and extensions may be found in the assigned textbook reading.



SOLID SPHERE

Mass point outside sphere ($r > R$)
A uniform solid sphere attracts an external mass point as if all the mass of the sphere is concentrated at its center.

$F_g = GMm/r^2$



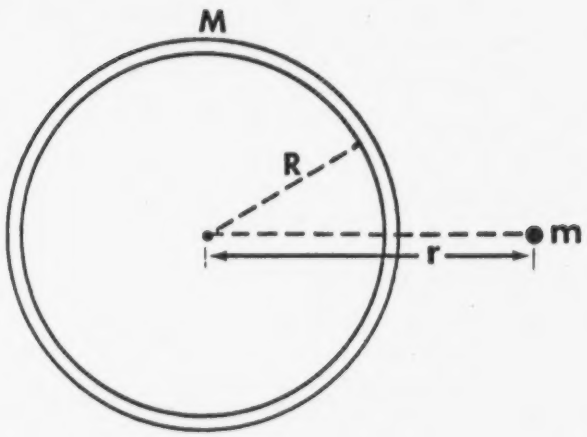
SOLID SPHERE

Mass point inside sphere ($R > r$)
A uniform solid sphere attracts an internal mass point as if all the particles lying outside the radial distance R of the mass point have zero mass, hence, no gravitational effect on the mass point. All the remaining mass is then considered to be concentrated at the center.

$F_g = GMm/r^2$

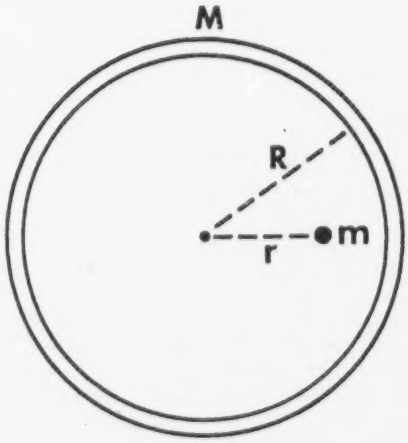
next page

continued



SPHERICAL SHELL
Mass point outside shell (r > R)
A uniform spherical shell attracts an external mass point as if all the mass of the shell is concentrated at its center.

$F_g = GMm/r^2$



SPHERICAL SHELL
Mass point inside shell (R > r)
The gravitational force exerted by a uniform spherical shell on an internal mass point is *zero*.

$F_g = 0$

Problems you will me include the

(a) determination of gravitational field strength as a function of distance from the center of a uniform spherical shell.

(b) writing of an expression for the gravitational force on a mass point lying inside a uniform shell which has a second mass point external to it.

19. Consider a sphere of radius R and total mass M, having uniform mass density. Calculate the gravitational field associated with this sphere as a function of r, the distance of the field point from the center of the sphere.

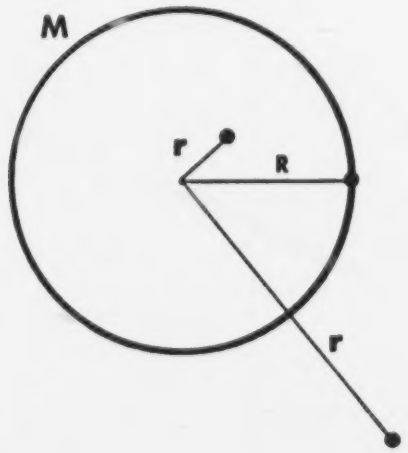
A. $\gamma = -\frac{GM}{r^2}$ everywhere

B. $\gamma = 0$ for $r < R$; $\gamma = -\frac{GM}{r^2}$ for $r > R$

C. $\gamma = -\frac{GM}{R^3} r$ for $r < R$; $\gamma = -\frac{GM}{r^2}$ for $r > R$

D. $\gamma = -\frac{GM}{R^2}$ for $r < R$; $\gamma = -\frac{GM}{r^2}$ for $r > R$

20. For the spherical shell of radius R and mass M in the accompanying diagram, what is the gravitational field strength as a function of distance from the center?



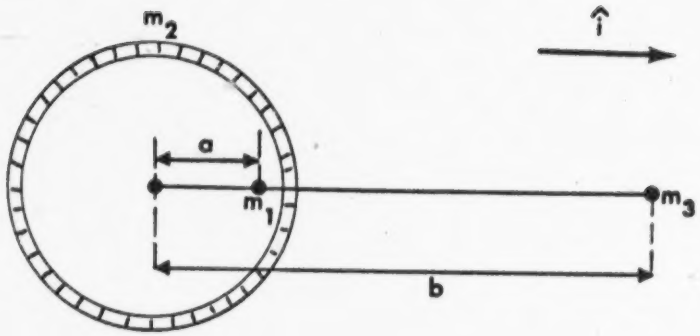
A. $\gamma = -\frac{GM}{r^2}$ everywhere

B. $\gamma = 0$ for $r < R$; $\gamma = -\frac{GM}{r^2}$ for $r > R$

C. $\gamma = \frac{GM}{r^2}$ for $r < R$; $\gamma = -\frac{GM}{r^2}$ for $r > R$

D. $\gamma = \frac{GM}{r^2}$ for $r > R$; $\gamma = -\frac{GM}{r^2}$ for $r < R$

21.



Consider the configuration shown above consisting of two particles of mass m_1 and m_3, and a shell of uniform mass density with total mass m_2. The distances of m_1 and m_3 from the center of the shell are a and b respectively. (The present information will be used in the next question.) What is the total gravitational force on m_1?

A. $-G \frac{m_1 m_2}{a^2} \hat{i}$

B. $G \frac{m_1 m_3}{(b - a)^2} \hat{i}$

C. $-G \left[\frac{m_1 m_3}{(b - a)^2} + \frac{m_1 m_2}{a^2} \right] \hat{i}$

D. zero

22. For the particles and spherical shell in the diagram of the previous problem, what is the total force on m_3?

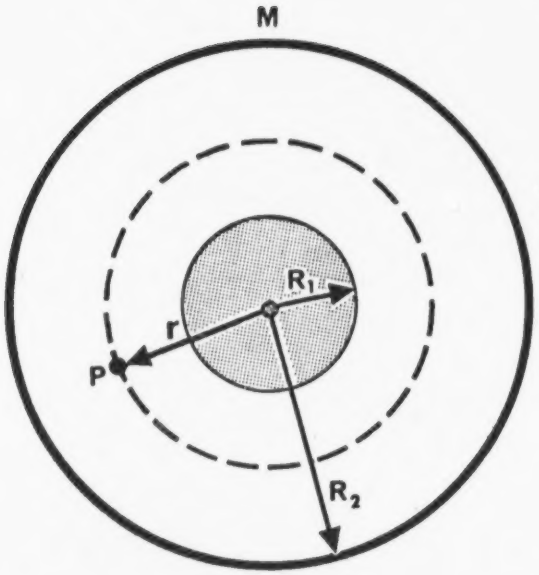
A. $- \frac{G m_2 m_3}{b^2} \hat{i}$

B. $- \frac{G m_1 m_3}{(b - a)^2} \hat{i}$

C. $-G \left[\frac{m_2 m_3}{b^2} + \frac{m_1 m_3}{(b - a)^2} \right] \hat{i}$

D. zero

23.



The figure shows a sphere of matter of radius R_2, and mass M, uniformly distributed. This sphere has a concentric cavity of radius R_1. Find the magnitude of the gravitational field as a function of r for the region $R_1 < r < R_2$.

A. $\dfrac{GM(R_1^3 - r^3)}{r^2}$

B. $\dfrac{GM(r^3 - R_1^3)}{r^2(R_2^3 - R_1^3)}$

C. $\dfrac{GMR_1^3}{r^2}$

D. $\dfrac{GMR_1^3}{r^2(R_2^3 - R_1^3)}$

[a] CORRECT ANSWER: 1.7 m/sec^2

The gravitational force on a particle of mass m located at the surface of the moon is equal to the weight of the mass, mg. Thus,

$$mg = F = G\frac{Mm}{R^2}$$

where M and R are the mass and radius of the moon, respectively.

Therefore,

$$g = \frac{GM}{R^2} = \frac{6.7 \times 10^{-11} \times 7.35 \times 10^{22}}{(1.7 \times 10^6)^2} = 1.7 \text{ m/sec}^2$$

[b] CORRECT ANSWER: B

The force exerted on a test particle by a spherical shell is zero inside the shell and the same as if the mass of the shell were concentrated at its center for test particles exterior to the shell.

So, for $r < R$,

$$\gamma = \frac{F}{m} = \frac{0}{m} = 0$$

and for $r > R$,

$$\gamma = \frac{F}{m} = \frac{1}{m}\left[\frac{-GMm}{r^2}\right] = -\frac{GM}{r^2}$$

[c] CORRECT ANSWER: A

Newton's law of universal gravitation is just that--universal. The gravitational force is proportional to the product of the masses of two particles and the inverse square of the distance between them; the constant of proportionality, G, is the same everywhere and for all time (as far as we know).

[a] CORRECT ANSWER: C

The first four equations are closely related. $\vec{F} = m\vec{a}$ is fundamental to what we mean by inertial mass. $F = mv^2/r$ is the same equation for the special case of uniform circular motion where

$$a = \frac{v^2}{r}$$

The momentum of a body $\vec{p} = m\vec{v}$ is related to $\vec{F}$ by Newton's second law,

$$F = \frac{d\vec{p}}{dt}$$

The expression for kinetic energy was derived as a force integral,

$$\Delta K = \vec{F} \cdot d\vec{s}$$

and does not depend upon gravitational attraction.

The remaining three equations (5, 6, and 7) and connected with Newton's law of universal gravitation

$$F = G \frac{m_1' m_2'}{r^2}$$

where the primed masses stand for gravitational mass. The important property of these gravitational masses is that they attract each other. Inertial mass, on the other hand, is a measure of resistance to being accelerated.

TRUE OR FALSE? The mass of a ball rotating in a horizontal circle on a string is determined experimentally using

$$m = \frac{Fr}{v^2}$$

The mass determined this way is the inertial mass of the ball.

[a] CORRECT ANSWER: 245 nt

The acceleration due to gravity is given in general by

$$g = G\frac{M_e}{r^2}$$

where a spherically symmetric Earth is assumed. At the surface of the Earth

$$g_o = G\frac{M_e}{R_e^2} \tag{1}$$

while an an altitude h

$$g = G\frac{M_e}{(R_e + h)^2} \tag{2}$$

But in our problem h = 6400 km = R_e, so (2) becomes, using (1)

$$g = G\frac{M_e}{(2R_e)^2} = G\frac{M_e}{4R_e^2} = \frac{1}{4}\left[G\frac{M_e}{R_e^2}\right] = \frac{g_o}{4}$$

Therefore, the acceleration of gravity at h = 6400 km is

$$g = \frac{9.8}{4}\ \text{m/sec}^2 = 2.45\ \text{m/sec}^2$$

and the weight of the 100-kg mass is

$$w = mg = 100 \times 2.45 = 245\ \text{nt}$$

[b] CORRECT ANSWER: D

All balances are thought of as measuring the weight of objects. This is not true. In spring balances the fact that springs obey Hooke's law, $F = -k(x - x_o)$, is used to measure the weight of bodies. Lever balances, on the other hand, compare the weight of a body with the weight of other bodies of known mass. Since both bodies are subjected to the same acceleration due to gravity, a lever balance in comparing weights, essentially compares the mass of an object with a standard mass. Since the mass of a body is a constant characteristic of the body, a lever balance would not register any difference in the mass ("weight") of the 70-kg mass no matter at what part of the Earth's surface the mass is.

[a] CORRECT ANSWER: C

The centripetal force necessary to cause a satellite to revolve about the Earth is the Earth's gravitational force. Thus,

$$\frac{mv^2}{(2R_e)} = \frac{GM_em}{(2R_e)^2} \qquad (1)$$

and

$$v^2 = \frac{GM_e}{2R_e} \qquad (2)$$

The acceleration due to gravity g near the surface of the Earth, is

$$g = \frac{GM_e}{R_e^2} \qquad (3)$$

But if we multiply equation (3) by $\frac{R_e}{2}$, it may be rewritten as

$$\frac{gR_e}{2} = \frac{GM_e}{2R_e} \qquad (4)$$

Combining equation (2) with equation (4), yields

$$v^2 = \frac{gR_e}{2}$$

$$v = \sqrt{\frac{gR_e}{2}}$$

TRUE OR FALSE? The velocity of the satellite described above varies directly as the radius of the Earth.

[a] CORRECT ANSWER: D

From Newton's Second Law, we have

$$\sum F_c = ma_R \qquad \text{where } F_c = \text{centripetal force}$$

The centripetal force *is* the force of gravity Fg (true weight).

Thus,

$$F_g - w_a = \frac{mv^2}{R}$$

where w_a is the *apparent* weight of the body at the equator.

Thus,

$$w_a = F_g - \frac{mv^2}{R} = \frac{GM_m}{R^2} - m\omega^2 R$$

but the weight at the pole $W = \frac{GM_m}{R^2}$ and $m = \frac{W}{g}$

so

$$w_a = W - \frac{W}{g}\omega^2 R = W\left[1 - \frac{\omega^2 R}{g}\right]$$

where g = gravitational acceleration near the surface.

TRUE OR FALSE? In this solution, it is not necessary to use the relation $\omega = 2\pi f$.

[b] CORRECT ANSWER: B

The force exerted by a spherical shell on a particle inside the shell is zero, but the spherical shell does not in any way interfere with the force exerted by other masses on the particle inside. So the total force on m_1 is that exerted by m_3 and is equal to

$$\frac{Gm_1m_3}{(b-a)^2}\,\vec{i}$$

in the positive x-direction

[a] CORRECT ANSWER: B

We may assume a sphere of uniform mass density to be made up of concentric shells. The contribution to the gravitational field at a point p at a distance r from the center of sphere is entirely due to the shells with radii less than r. The contribution of the shells with radii greater than r to the gravitational field at p is zero. The mass of the sphere of radius r with the concentric cavity of radius R_1 is

$$M' = \frac{4}{3}\pi\rho(r^3 - R_1{}^3) \tag{1}$$

where ρ is the uniform mass density

$$\rho = \frac{M}{V} = \frac{M}{\frac{4\pi}{3}(R_2{}^3 - R_1{}^3)} \tag{2}$$

Substituting ρ from (2) in (1), we get

$$M' = \frac{M(r^3 - R_1{}^3)}{(R_2{}^3 - R_1{}^3)}$$

Thus the magnitude of the gravitational field in the region $R_1 < r < R_2$ is

$$\gamma = \frac{GM'}{r^2} = \frac{GM(r^3 - R_1{}^3)}{r^2(R_2{}^3 - R_1{}^3)}$$

TRUE OR FALSE? The center of mass of the body described above lies outside the cavity.

[b] CORRECT ANSWER: C

The shell acts on a particle exterior to it as though its mass were concentrated at its center, which is a distance b from m_3. The distance of m_1 from m_3 is (b - a), so the total force on m_3 is

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} = -G\frac{m_2m_3}{b^2}\hat{i} - G\frac{m_1m_3}{(b-a)^2}\hat{i}$$

$$= -Gm_3\left(\frac{m_2}{d^2} + \frac{m_1}{(b-a)^2}\right)\hat{i}$$

[a] CORRECT ANSWER: $11.2\ m/sec^2$

The gravitational field $\vec{\gamma}$ due to a massive body is the gravitational force on a unit mass. We may write

$$\vec{\gamma} = \vec{F}/m$$

where $\vec{F}$ is the force on a particle of mass m due to the massive body. In this case,

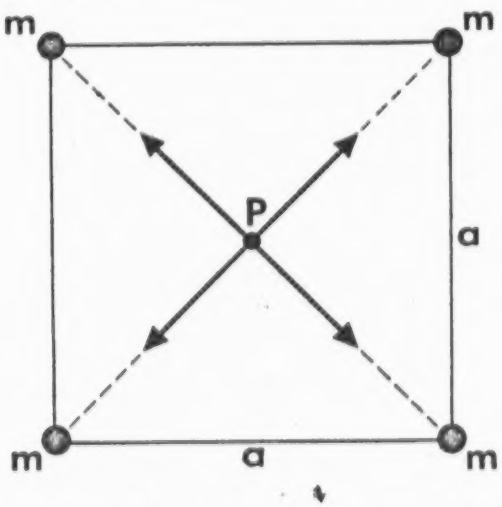
$$\gamma = GM/R^2$$

where M is the mass of Saturn and R is its radius.

Substituting the given numerical data,

$$\gamma = \frac{6.67 \times 10^{-11} \times 5.7 \times 10^{26}}{(5.83 \times 10^7)^2} = 11.2\ m/sec^2$$

[b] CORRECT ANSWER: A



The magnitude of the field produced by a mass m at any point a distance r away is given by

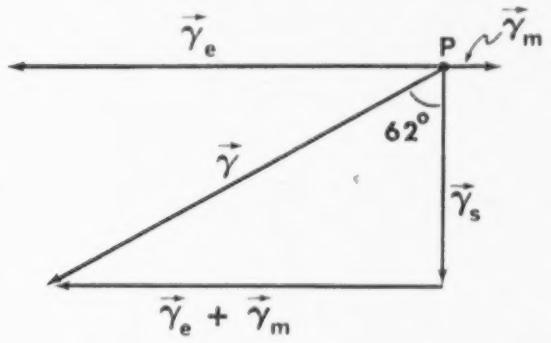
$$\gamma = \frac{GM}{r^2}$$

In the present case, there are contributions to the gravitational field at P due to four masses. These contributions must be added vectorially to obtain the resultant field. Since the contributions from the masses which are placed diagonally opposite to each other are equal in magnitude but exactly oppsite in direction the total field at P is zero.

TRUE OR FALSE? The magnitude of the contribution to the gravitational field at P of each of the four masses is the same.

[a] CORRECT ANSWER: A

The magnitude of the field produced by a mass m at any point a distance r away is given by



$$\gamma = \frac{GM}{r^2}$$

In the present situation we have three contributions; namely, from the three celestial bodies involved. These contributions must be added vectorially since $\vec{\gamma}$ is a vector quantity. Thus, at point P

$$\vec{\gamma} = \vec{\gamma}_e + \vec{\gamma}_m + \vec{\gamma}_s$$

or

$$\vec{\gamma} = G\left[\frac{M_e}{(r_{em}/2)^2}(-\hat{i}) + \frac{M_m}{(r_{em}/2)^2}(\hat{i}) + \frac{M_s}{(r_s)^2}(-\hat{j})\right]$$

$$= -G\left[\frac{M_e - M_m}{(r_{em}/2)^2}\hat{i} + \frac{M_s}{(r_s)^2}\hat{j}\right]$$

$$= -6.67 \quad 10^{-11}\,\frac{5.93 \times 10^{24}}{(1.9 \times 10^{8})^2}\hat{i} + \frac{2 \times 10^{30}}{(1.5 \times 10^{11})^2}\hat{j}$$

$$= -6.67 \quad 10^{-11}\,(1.65 \times 10^{8}\,\hat{i} + 0.89 \times 10^{8}\,\hat{j})$$

$$= -(11\,\hat{i} + 5.9\,\hat{j}) \times 10^{-3} \text{ nt/kg}$$

The magnitude of this vector

$$\gamma = \sqrt{(11)^2 + (5.9)^2} \times 10^{-3} \text{ nt/kg} = 1.25 \times 10^{-2} \text{ nt/kg}$$

The angle that this vector makes with the "negative y-axis" is

$$\theta = \tan^{-1}\frac{11}{5.9} = \tan^{-1}(1.87) = 62°$$

TRUE OR FALSE? Problems of this type require that you recognize the scalar nature of γ.

[a] CORRECT ANSWER: B

If a is the centripetal acceleration of Phobos provided by Mars' gravitational attraction and r is its distance from the center of Mars, the force exerted by Mars on a mass m at a distance r from the surface is

$$ma = \frac{GmM}{r^2} \tag{1}$$

The force exerted by Mars on the same mass m near its surface on the same mass m near its surface is given by

$$mg = G\,\frac{mM}{R^2} \tag{2}$$

Taking the ratio of equation (2) to (1), we have

$$\frac{a}{g} = \left(\frac{R}{r}\right)^2 \tag{3}$$

The mass m in this case is that of Phobos which is in circular orbit around Mars. Hence, the acceleration a is the centripetal acceleration and is equal to v^2/r, where v is Phobos' tangential velocity:

$$a = \frac{v^2}{r} = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2} \tag{4}$$

Substitution of equation (4) in equation (3) yields

$$\frac{1}{g}\left[\frac{4\pi^2 r}{T^2}\right] = \left[\frac{R}{r}\right]^2$$

or

$$r^3 = g\left[\frac{TR}{2\pi}\right]^2 = 3.8 \times \left[\frac{2.76 \times 10^4 \times 3.4 \times 10^6}{2\pi}\right]^2 = 8.50 \times 10^{20}\ m^3$$

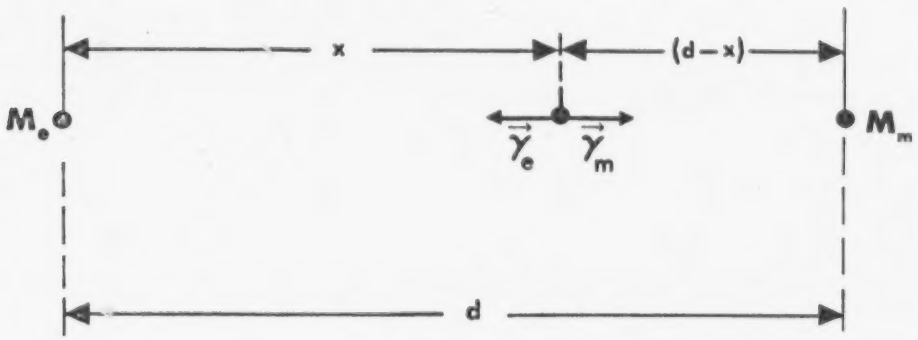
So

$$r = 9{,}500\ km$$

TRUE OR FALSE? The mass of Phobos in this solution is symbolized by M.

[a] CORRECT ANSWER: A

We want to find the point at a distance x from the center of the Earth at which the Earth's field strength $\vec{\gamma}_e$ is equal in magnitude and opposite in direction to the field strength $\vec{\gamma}_m$ of the moon m; i.e., $\vec{\gamma}_e = -\vec{\gamma}_m$. (See diagram.)



Setting $\gamma_e = \gamma_m$ we obtain

$$\frac{GM_e}{x^2} \quad \frac{GM_m}{(d - x)^2}$$

which gives

$$x = \sqrt{\frac{M_e}{M_m}}\,(d - x)$$

or

$$x\left[1 + \sqrt{\frac{M_e}{M_m}}\right] = \sqrt{\frac{M_e}{M_m}}\,d$$

and

$$x = \frac{d\sqrt{M_e/M_m}}{1 + \sqrt{M_e/M_m}} \qquad (1)$$

next page

continued

From the given data we find

$$\sqrt{\frac{M_e}{M_m}} = \sqrt{\frac{6 \times 10^{24}}{7.3 \times 10^{22}}} = \sqrt{82.2} = 9.06$$

Substituting in (1) we obtain

$$x = \frac{3.8 \times 10^8 \times 9.06}{1 + 9.06} = 3.4 \times 10^8 \text{ m}$$

[a] CORRECT ANSWER: C

We apply Newton's law of universal gravitation to mass m at the surface of Mars,

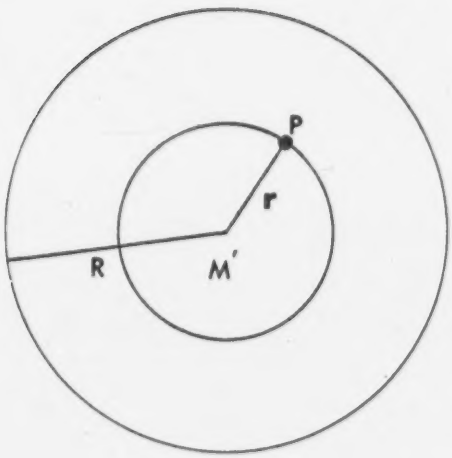
$$mg = F = \frac{GMm}{R^2}$$

to find

$$M = \frac{gR^2}{G} = \frac{3.8 \times (3.4 \times 10^6)^2}{6.7 \times 10^{-11}} = 6.55 \times 10^{23} \text{ kg}$$

The data concerning Phobos is irrelevant to this problem.

[a] CORRECT ANSWER: C



We can imagine a sphere of uniform mass density as made up of concentric shells each with the same mass density. The contribution to the field at a point P a distance r from the center of the sphere, of all the shells with radii greater than r, is zero. The remainder of the original sphere which contributes to the field at P has a mass

$$M' = \rho\left[\frac{4}{3}\ \pi r^3\right] \qquad (1)$$

But, since the total mass of the uniform sphere is M we have

$$\rho = \frac{M}{V} = \frac{M}{(4/3)\ \pi R^3} \qquad (2)$$

and (1) becomes

$$M' = \frac{M(4/3)\ \pi r^3}{(4/3)\ \pi R^3} = M(r/R)^3 \qquad (3)$$

Thus, the gravitational field for $r < R$ is

$$\gamma_{r<R} = -G\,\frac{M'}{r^2} = -G\,\frac{Mr^3}{R^3 r^2} = -G\,\frac{Mr}{R^3} \qquad (4)$$

next page

continued

For points outside the sphere ($r > R$), of course, the field is the same as if all of the mass of the sphere were concentrated at its center. Thus, for $r > R$ the field is

$$\gamma_{r>R} = -G \frac{M}{r^2} \qquad (5)$$

Note the continuity of the gravitational field at $r = R$. That is, note that the two expressions (4) and (5) become the same for $r = R$, namely

$$\gamma_{r=R} = -G \frac{M}{R^2}$$

TRUE OR FALSE? If a given shell has a radius larger than r, then this shell contributes nothing to the gravitational field at P.

[a] CORRECT ANSWER: 2.03×10^{20} nt

The gravitational force between two bodies separated by distance r is given by

$$F = G \frac{m_1 m_2}{r^2}$$

Thus, the gravitational force between the moon and the Earth is

$$F = G \frac{M_e M_m}{r_{em}^2} \quad \frac{6.7 \times 10^{-11} \times 6 \times 10^{24} \times 7.3 \times 10^{22}}{(3.8 \times 10^8)^2}$$

$$= 2.03 \times 10^{20} \text{ nt}$$

TRUE OR FALSE? In solving this problem, the assumption is made that the Earth and moon are perfect spheres with symmetric mass distributions.

[a] CORRECT ANSWER: C

We apply HR equation 16-10,

$$r^3 = \frac{GM_eT^2}{4\pi^2}$$

which relates the period and radius of circular satellite orbits about the Earth. If the satellite is always to remain directly above a certain point on the Earth's equator, the satellite must have the same period as the Earth. The Earth's rotation period with respect to the sun is, of course, 24 hours, but here we must use the sidereal day (rotation period of the Earth with respect to the "fixed stars"), which is approximately 23 hours 56 minutes. To derive this period, recall that there are 366.25 sidereal days in the year. In the process of making one revolution around the sun, the Earth rotates once with respect to the fixed stars. Thus the sidereal day is (1/365) × 24 hours shorter than the mean solar day. This is approximately 4 minutes (236 seconds), so the period we need is (86,400 - 236) sec = 86,164 sec. Thus

$$r^3 = \frac{GM_eT^2}{4\pi^2} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (8.62 \times 10^4)^2}{4\pi^2}$$

from which r = 42,000 km. Subtract off the radius of the earth (6,400 km) to obtain the satellite's altitude of 35,600 km.

[b] CORRECT ANSWER: False *(decreases)*

We get the acceleration of gravity, g, by using Newton's second law of motion, and his law of universal gravitation. Thus, assuming a spherically symmetric Earth,

$$mg = F = G\frac{mM}{r^2}$$

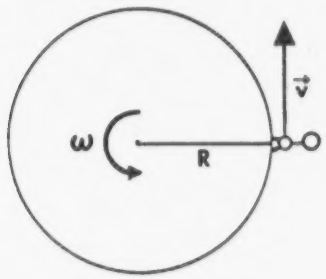
where M is the mass of the Earth and r is the distance from the Earth's center. It follows that

$$g = G\frac{M}{r2}$$

So as r increases, g decreases.

[a] CORRECT ANSWER: C

The average difference in the value of the acceleration due to gravity between the poles and the equator of our "imperfectly" spherical Earth is 0.0518 m/sec^2. This difference results from the combination of the Earth's *rotation* about its axis and its equatorial bulge. The effect of the Earth's rotation is what we must compute, since we eliminated the equatorial bulge effect by assuming a perfectly spherical Earth.



Looking down toward the Earth from a point in space directly above the north pole, an observer would see it rotating from west to east with an angular speed

$$\omega = \frac{2\pi}{T} \qquad (1)$$

where T is the period of rotation; namely, T = 86,400 sec.

If the Earth's gravitational attraction was suddenly "turned off", all objects not attached to the Earth by other than gravitational forces would go flying in a direction tangent to the circular path they follow in space. In order for the m = 70-kg man to remain on the equator, following the Earth in its rotation, a centripetal force

$$F_c = \frac{mv^2}{R} = m\omega^2 R = \frac{4\pi^2 mR}{T^2} \qquad (2)$$

would be required. No such force is required to keep this man on the poles of the Earth, since at that point the mass would be stationary if the only motion involved were the Earth's rotation about its axis.

Now, a perfectly spherical Earth attracts all bodies having mass m with a force

$$F_g = G\,\frac{mM}{R^2} \qquad (3)$$

At the poles of the Earth, a spring balance will register the full gravitational force F_g. At the equator, however, part of F_g would be required to provide the centripetal acceleration so the balance will register an apparent weight

$$w_a = F_g - F_c \qquad (4)$$

next page

continued

The balance, therefore, will read more at the poles. This difference in the apparent weight is given by (2) and is:

$$\Delta w_a = F_c = \frac{4\pi^2 \times 70 \times 6.4 \times 10^6}{(8.64)^2 \times 10^8} = 2.36 \text{ nt}$$

You may be wondering why we must consider a "massless" spring balance. If the balance were not massless, the weight of the balance--whether it is of the hanging spring type or the platform type--would result in some reading. This is taken into account by calibrating the balance so that it reads zero when no other masses are attached to it. Since, however, the balances own apparent weight would change when it is taken from the equator to the poles, a balance calibrated at the equator would not be suitable at the poles. A new calibration is required there. A "massless" spring balance avoids all these complications.

Up to this point, of course, we have not made any distinction between the apparent and the actual weight of objects. When we equate the gravitational force on a body to the product mg, we are apparently neglecting all effects due to the rotation and the non-sphericity of the Earth. When we compare the apparent weight of the 70-kg man,

$$w_a \approx 70 \times 9.8 = 686 \text{ nt}$$

to equation (5) we see that the effect of rotation is less than 0.4%; i.e., may very well be neglected for most practical purposes. Even the actual maximum variation of the man's weight, $70 \times 0.0518 = 3.62$ nt, is about 0.5% of the average apparent weight and, therefore, can be neglected.

TRUE OR FALSE? If a "massless" spring balance had not been stipulated, the problem would have been insoluble even if the mass of the balance were given.

[a] CORRECT ANSWER: 3.6×10^{-47} nt

Newton's law of universal gravitation states that the force $\vec{F}_g$ due to gravitational attraction has a magnitude given by

$$F_g = \frac{Gm_1m_2}{r^2}$$

where m_1 and m_2 are the interacting masses, and r is the distance between the point masses. Therefore, the force exerted by the proton on the electron has a magnitude

$$F_g = \frac{Gm_em_p}{r^2} = \frac{6.7 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.7 \times 10^{-27}}{(5.3 \times 10^{-11})^2}$$

$$= 3.6 \times 10^{-47} \text{ nt}$$

The orbital period of the electron is irrelevant to the calculation.

TRUE OR FALSE? The gravitational force exerted by the *electron on the proton* is 3.6×10^{-47} nt.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Gravitational Potential Energy

OBJECTIVE

To make use of the concept of gravitational potential energy in the solution of problems for masses in a gravitational field.

In general, we are primarily concerned with *changes* in the potential energy of a body when work is done on it or by it. For convenience, we assume a given position as the zero reference level in problem applications where we are to determine the final potential energy of a body or a system. When objects are lifted or lowered over relatively short distances near the Earth's surface, the change in the gravitational force is small enough to be ignored. For situations like this, the zero reference level is taken as the Earth's surface. Hence, when work is done *on* a mass to lift it from the Earth, its potential energy of raised position increases in the positive direction. The gravitational potential energy of the body is then U = mgy where y is the upward displacement from the surface.

The restrictions regarding proximity to the surface of the Earth or small displacements cannot be retained in precise work; the force of gravity *does* change because it is a function of separation distance between the centers of gravitating masses. From this point of view, it is more sensible to consider the zero reference level to be located at the so-called "force-free" position which, for gravitation, is an infinite distance from the center of attraction.

The gravitational potential energy of a point mass is considered to be zero when it is separated from the center of attraction by an infinite distance.

To find the potential energy of a particle at some distance r from the attraction center, we must calculate the work done *on* the particle in moving at from infinity to its new position. That is,

$$U(r) = W_{\infty r} = \int_{\infty}^{r} F_g \, dr$$

next page

continued

Since $F_g = \frac{GMm}{r^2}$, then

$$U(r) = \int_{\infty}^{r} \frac{GMm}{r^2}\, dr = GMm \int_{\infty}^{r} \frac{1}{r^2}\, dr$$

so $$U(r) = GMm\left(-\frac{1}{r}\right)\Bigg|_{\infty}^{r}$$

and $$U(r) = -\frac{GMm}{r}$$

The significance of the negative sign should be heavily emphasized. The work on the particle would have been positive *if it were being further separated from the attracting center*, that is, if the work had been done *against* the attracting force. But in this action, the displacement was toward the attracting center, *with* the attracting force, hence the work is negative and so is the potential energy of the particle in the new position as compared with its original position at infinity. In other words

> *the potential energy of the particle decreases from zero to some negative value as it moves from infinity to a finite distance r from the attracting center.*

Problems involving gravitational potential energy require that you be capable of

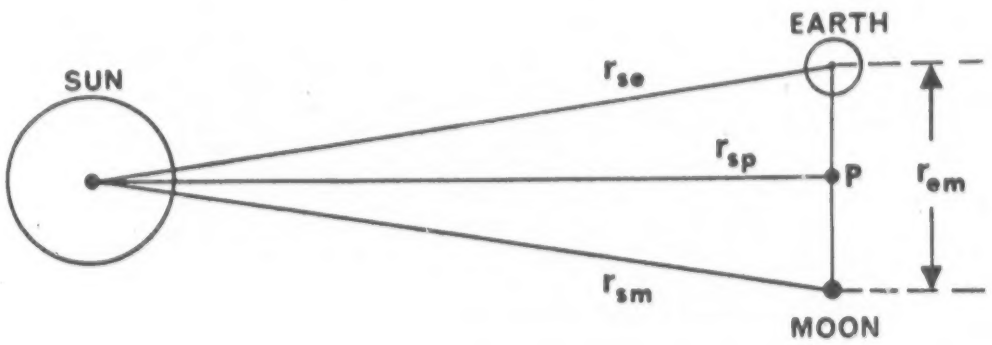
(a) determining U for a given mass located at a given distance from one or more centers of attraction;

(b) calculating work done against (or by) gravity, given displacements and masses.

(c) recognizing and utilizing the fact that potential energy is a *scalar* quantity.

PROBLEMS

1.



Referring to the diagram, what is the work required to bring a spaceship of mass m to the position P (halfway between the Earth and moon) from infinity? Assume that the Earth, moon, and sun are stationary.

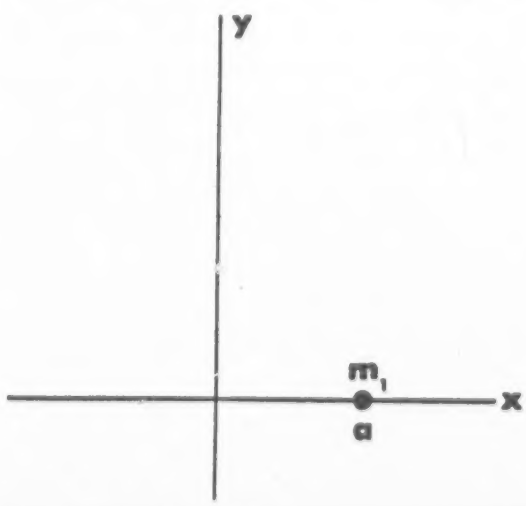
A. $-Gm\left(\frac{M_s}{r_{sp}} + \frac{M_e}{r_{em}} + \frac{M_m}{r_{em}}\right)$

B. $-Gm\left(\frac{M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{2M_m}{r_{em}}\right)$

C. $-Gm\left(\frac{2M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{2M_m}{r_{em}}\right)$

D. $-Gm\left(\frac{2M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{M_m}{r_{em}}\right)$

2. Neglecting the effect of *all* other masses in the universe, how much work must be done on a mass m_1 in moving it from $x = \infty$ to $x = a$, along the x-axis?



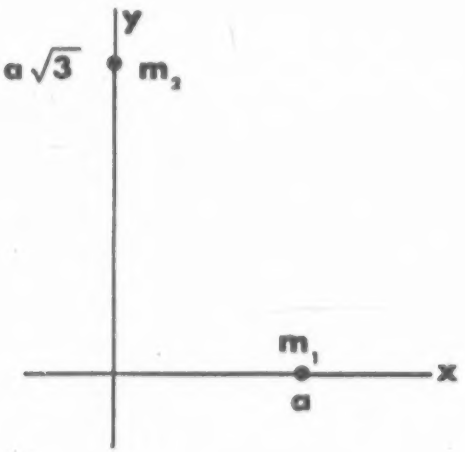
A. Zero

B. $-\frac{GM_1}{a}$

C. $\frac{GM_1}{a}$

D. Infinite

3. How much work is done when a mass m_1 is held stationary at point (a, 0) while mass m_2 is moved from point (0, $-\infty$) to point (0, $a\sqrt{3}$) along the y-axis? Ignore all other masses.



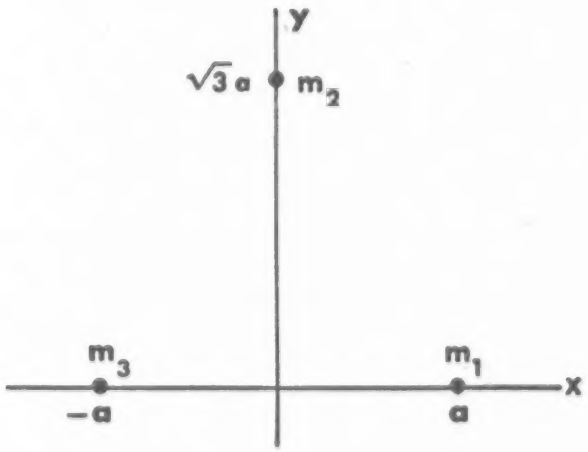
A. $-\frac{Gm_1m_2}{a\sqrt{3}} - \frac{Gm_1m_2}{a}$

B. $-\frac{Gm_1m_2}{a\sqrt{3}}$

C. $\frac{Gm_1m_2}{a\sqrt{3}} - \frac{Gm_1m_2}{a}$

D. $-\frac{Gm_1m_2}{2a}$

4. If masses m_1 and m_2 as shown in the figure are held stationary, while mass m_3 is moved from point ($-\infty$, 0) to point (-a, 0), along the x-axis, how much work needs to be done? (Neglect all other masses.)



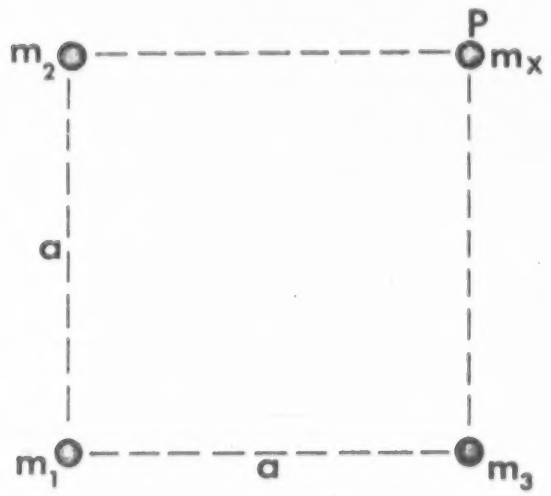
A. $-\frac{Gm_1m_3}{2a} - \frac{Gm_1m_2}{2a}$

B. $-\frac{Gm_1m_3}{2a} - \frac{Gm_2m_3}{2a}$

C. $-\frac{Gm_1m_3}{2a} - \frac{Gm_1m_2}{2a} - \frac{Gm_2m_3}{2a}$

D. $-\frac{Gm_1m_3}{2a} - \frac{Gm_2m_3}{2a} + \frac{Gm_1m_2}{2a}$

5. Originally three particles with masses m_1, m_2 and m_3 are situated at the three corners of a square with side of length a. Now a fourth particle with mass m_x is brought into the fourth corner of the square while the other three particles are held stationary. What is the change in the potential energy of the system?



A. $-\frac{Gm_1m_x}{a} - \frac{Gm_2m_x}{\sqrt{2a}} - \frac{Gm_3m_x}{\sqrt{2a}}$

B. $-\frac{Gm_1m_x}{\sqrt{2a}} - \frac{Gm_2m_x}{2a} - \frac{Gm_3m_x}{2a}$

C. $-\frac{Gm_1m_x}{\sqrt{2a}} - \frac{Gm_2m_x}{a} - \frac{Gm_3m_x}{a}$

D. $-\frac{Gm_1m_x - Gm_2m_x - Gm_3m_x}{a}$

INFORMATION PANEL — Gravitational Potential

OBJECTIVE

To clarify and use the concept of gravitational potential where this concept facilitates problem solving.

In your work with the gravitational field, you recognized that gravitational field strength γ is defined as gravitational force per unit mass, or $\gamma = \vec{F}/m$. What may need reemphasis here is that the *strength of a field is a property of the space in which the field appears, not of the mass particles that happen to be located in that space.*

This idea can be carried fruitfully into the sphere of gravitational potential energy. Consider three different mass particles that have been brought from infinity down to exactly the same altitude above the earth. The potential energies of these masses are, of course, all different, that is

$$U_1 = -\frac{GMm_1}{r} \qquad U_2 = -\frac{GMm_2}{r} \qquad U_3 = -\frac{GMm_3}{r}$$

because $m_1 \neq m_2 \neq m_3$.

next page

continued

However, we can now define a quantity which we shall call gravitational potential V. *Gravitational potential is gravitational potential energy per unit mass* or

$$V = \frac{U}{M}$$

Consistent with this definition, the gravitational potentials associated with the three mass particles discussed above are:

$$V_1 = -\frac{GMm_1}{rm_1} = -\frac{GM}{r}$$

$$V_2 = -\frac{GMm_2}{rm_2} = -\frac{GM}{r}$$

$$V_3 = -\frac{GMm_3}{rm_3} = -\frac{GM}{r}$$

and it is at once seen that the gravitational potential at a given point in a given field is the same regardless of the mass of the particle at that point.

> *Gravitational potential is a property of the space surrounding a body of matter and not of any mass particles that happen to be present in that space.*

Gravitational potential is a scalar quantity so that by the definition:

$$V = -GM/r$$

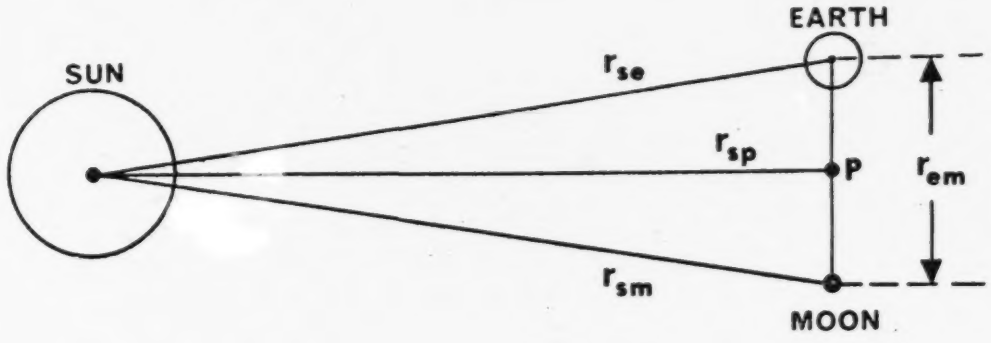
we have associated with gravitation a *scalar* field.

Furthermore, if we know the potential V at any point in space we can now determine the potential energy U of a mass particle m at that point simply by multiplying V by m.

In the MKS system, gravitational potential V is measured in joules per kilogram and in the British system in foot pounds per slug.

The problems that follow provide practice in calculating potential energies and gravitational forces utilizing the concept of gravitational potential, and vice versa.

6. What is the gravitational potential at the point P (halfway between the Earth and moon) in the diagram?



A. $-G\left[\frac{M_s}{r_{sp}} + \frac{M_e}{r_{em}} + \frac{M_m}{r_{em}}\right]$

B. $-G\left[\frac{M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{2M_m}{r_{em}}\right]$

C. $-G\left[\frac{2M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{2M_m}{r_{em}}\right]$

D. $-G\left[\frac{2M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{M_m}{r_{em}}\right]$

7. Which of the following pairs of statements correctly relates gravitational potential V and gravitational field strength γ for the special case of spherical symmetry?

A. $V(r) = \int_{\infty}^{r} \gamma(r)\, dr$ and $\gamma(r) = \frac{d}{dr} V(r)$

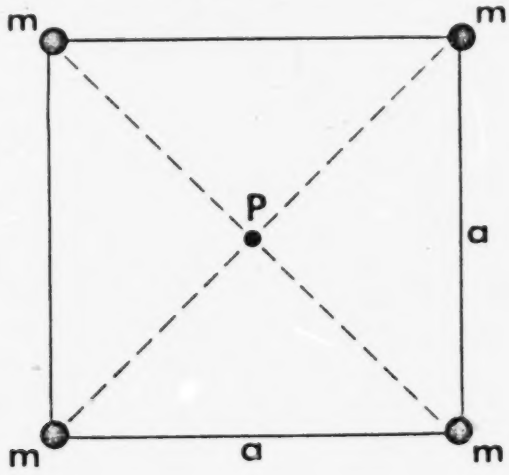
B. $V(r) = -\int_{\infty}^{r} \gamma(r)\, dr$ and $\gamma(r) = -\frac{d}{dr} V(r)$

C. $V(r) = -\int_{\infty}^{r} \gamma(r)\, dr$ and $\gamma(r) = \frac{d}{dr} V(r)$

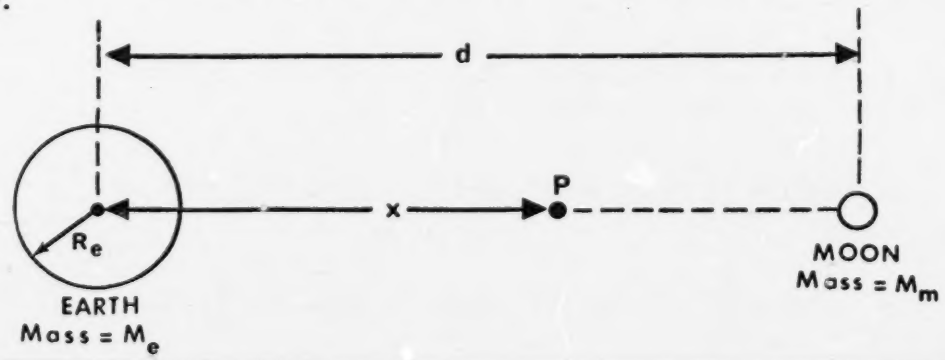
D. $V(r) = \int_{\infty}^{r} \gamma(r)\, dr$ and $\gamma(r) = -\frac{d}{dr} V(r)$

8. Find an expression for the gravitational potential V at a point which lies along a line joining the centers of the Earth and moon. The point is a distance x from the Earth's center, the distance between Earth and moon is d, and the respective masses of Earth and moon are M_e and M_m.

9. Four equal masses (each of mass m) are placed at the corners of a square with sides of length a. Neglecting the effect of all other masses in the universe, find the gravitational potential V at point P, the geometric center of the square.



10. In the figure below, the gravitational field at point P is zero; hence, a spaceship placed there would experience no force and would remain there indefinitely if the Earth-moon system were a rigid body. (A rigid body is a body whose parts have a fixed location with respect to each other.) Suppose that the Earth-moon system were a rigid body and that the effects of all other celestial bodies were negligible; with what speed would a rocket, aimed directly toward the moon, have to leave the surface of the Earth in order that it would be "captured" at point P. Express your answer in terms of the symbols in the diagram below.



11. The total kinetic and mechanical energies (K and E, respectively) of a satellite in circular orbit about the Earth are related to the potential energy U of the system by

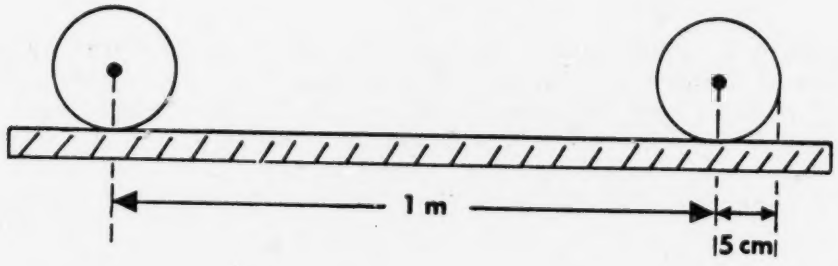
A. $K = U; \quad E = 2U$

B. $K = -U; \quad E = 0$

C. $K = -(1/2)U; \quad E = (1/2)U$

D. $K = (1/2)U; \quad E = (3/2)U$

12. Two apples are held at rest 1 m apart on a horizontal frictionless table. Assuming the apples to be uniform spheres each of radius 5 cm and mass 0.2 kg, what will be their speed relative to the table when they collide some time after they are released?



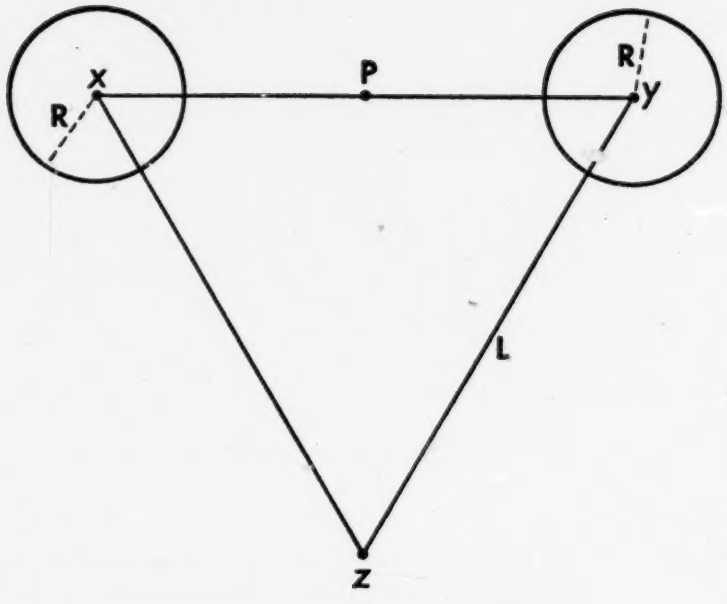
A. 3.45×10^{-5} m/sec

B. 1.55×10^{-5} m/sec

C. 1.1×10^{-5} m/sec

D. 2.44×10^{-5} m/sec

13.



In the figure x, y and z are the vertices of an equilateral triangle with sides of length L. Points x and y represent the centers of two identical planets each of mass M and radius R. With what speed should a spaceship leave the point P, equidistant from x and y, so as to reach point z with zero kinetic energy? (Ignore the effects of all other celestial bodies.) Express your answer for v in terms of G, M, and L.

INFORMATION PANEL Escape Speed

OBJECTIVE

To recognize and use the energy relationships required for the determination of the escape speed of a body from a planet.

The potential energy of a point mass, as you have seen, is given by

$$U(r) = -\frac{GMm}{r} \qquad (1)$$

where M = mass of the center of attraction; m = point mass; and r = distance between the point mass and the center of attraction. This equation may be rewritten for a particle on the Earth's surface as

$$U(r) = -\frac{GM_e m}{R_E} \qquad (2)$$

since m is separated from the center of the Earth by an amount equal to the Earth radius R_e. Now imagine that the particle is projected straight up with sufficient kinetic energy to enable it to travel to infinity before it comes to rest. After projection, only the gravitational force between the particle and the Earth is to be considered effective, hence, the system is conservative. This, in turn, means that all of the initial kinetic energy must have been used to produce the net change in potential energy or

$$\frac{1}{2} mv_o^2 = U_\infty - U_{RE} = 0 - \left(-\frac{GM_E m}{R_e}\right)$$

or

$$\frac{1}{2} mv_o^2 = \frac{GM_E m}{R_e} \qquad (3)$$

The particle *gains* potential energy in going from the Earth's surface (a point of *negative* potential energy) to infinity where its potential energy is zero, hence, the positive sign before $GM_e m/R_e$ in equation (3).

From a slightly different point of view it is clear that the total energy of the particle-Earth system is zero when the particle reaches infinity because

(a) it has stopped moving so $K_f = 0$ and

(b) it has reached the zero reference level so $U_f = 0$

next page

continued

Therefore

$$K_f + U_f = 0$$

but since only conservative forces are at work, the initial energy of the system must have been zero too, so

$$K_i + U_i = 0$$

or

$$\frac{1}{2} mv_o^2 + \left(- \frac{GM_{Em}}{R_E} \right) = 0$$

so

$$\frac{1}{2} mv_o^2 - \frac{GM_{E}m}{R_E} = 0 \qquad (4)$$

The escape speed v_o is the initial speed which makes the initial total energy equal zero.

From either equation (3) or (4) we have

$$v_o = \sqrt{\frac{2GM_e}{R_e}} = 6.96 \text{ mi/sec}$$

$$= 25{,}000 \text{ mi/hr}$$

$$= 11.2 \text{ km/sec}$$

You will be expected to solve problems typified by

(a) determination of the initial altitude for which the escape speed would be a given figure for a given planet;

(b) calculation of v_o from a planet such as Mars;

(c) calculation of the escape speed, given the distance from and the mass of the parent body.

14. At what altitude above the Earth's surface is the escape velocity (speed) from the Earth equal to 10 km/sec? (Take the Earth's radius equal to 6400 km and its mass equal to 6×10^{24} kg.)

15. The planet Mars has a mass of about 6×10^{23} kg and a radius of 3.5×10^{6} m. Calculate the *escape velocity* (speed) of a 25-kg body from the surface of Mars.

16. What is the escape speed from the sun at the position of the Earth's orbit; i.e., about 1.5×10^{11} m away from the sun? (The mass of the sun is 2×10^{30} kg.)

[a] CORRECT ANSWER: $V = -G\left(\frac{M_e}{x} + \frac{M_m}{d - x}\right)$

The contribution of the Earth to the gravitational potential at a distance x from its center is

$$V_e = -GM_e/x$$

Similarly, the moon's contribution to the gravitational potential at a distance y from its center is

$$V_m = -GM_m/y$$

Since the point in which we are interested lies along the line joining the centers of the two bodies,

$$x + y = d$$

Therefore, the total gravitational potential at this point is

$$V = -G\left(\frac{M_e}{x} + \frac{M_m}{(d - x)}\right)$$

[b] CORRECT ANSWER: 4.8 km/sec

We can use conservation of energy to solve this problem.

$$-G\,\frac{mM}{R_m} + \frac{1}{2}\,mv_i^2 = -\,\frac{GmM}{R_m} + \frac{1}{2}\,mv_f^2 \qquad (1)$$

(M and R_m are Mars' mass and radius, respectively.) By definition the escape speed is the speed which will be required to just get the object out of the gravitational influence of Mars. The object will have no velocity at an infinite point ($R = \infty$), and the final energy is zero. Therefore, by conservation of energy the initial total energy must also be zero.

$$-\,\frac{GmM}{R_m} + \frac{1}{2}\,mv_i^2 = 0 \qquad (2)$$

Solving (2) for v_i, we obtain

$$v_i = \sqrt{\frac{2Gm}{R_m}} = \sqrt{\frac{2 \times 6.7 \times 10^{-11} \times 6 \times 10^{23}}{3.5 \times 10^6}}$$

$$= 4.8 \times 10^3 \text{ m/sec} = 4.8 \text{ km/sec}$$

This is the escape speed; i.e., the minimum speed which a particle must have in order to escape the gravitational field of Mars.

[a] CORRECT ANSWER: C

The total potential energy of the system is simply the sum of the potential energy between any two masses. Therefore, the potential energy of the original system is

$$U = U_{12} + U_{23} + U_{13}$$

The total potential energy of the new system is

$$U' = U_{12} + U_{23} + U_{13} + U_{1x} + U_{2x} + U_{3x}$$

Thus the change in the potential energy is

$$\Delta U = U' - U = U_{1x} + U_{2x} + U_{3x}$$

$$= -\frac{Gm_1m_x}{\sqrt{2a}} - \frac{Gm_2m_x}{a} - \frac{Gm_3m_x}{a}$$

TRUE OR FALSE? The potential energy of the partial system consisting of m_2 and m_x is

$$-\frac{Gm_2m_x}{\sqrt{2a}}$$

[b] CORRECT ANSWER: 1,600 km

The escape velocity (speed) is the speed which makes the total energy of a particle equal to zero, or its kinetic energy equal to the negative of its potential energy,

$$\frac{1}{2}mv_o^2 = -\left[-\frac{GM_em}{r}\right]$$

Solving for r, the distance from the Earth's center,

$$r = \frac{2GM_e}{v_o^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(10)^2} = 8 \times 10^6 \text{ m}$$

Thus, the altitude above the Earth's surface is

$$h = r - R_e = 8{,}000 \text{ km} - 6{,}400 \text{ km} = 1{,}600 \text{ km}$$

TRUE OR FALSE? The required altitude is inversely proportional to the escape speed.

[a] CORRECT ANSWER: 42 km/sec

The initial energy E_i of a mass m which is about to escape infinitely far from the sun's attraction is given by

$$E_i = K_i + U_i$$

$$= \frac{1}{2} mv_e^2 - \frac{GM_s m}{r}$$

where r is the initial distance from the sun. The minimum final energy E_f of the escaped mass is zero because $U_f = 0$ at an infinite distance from the sun, and the least kinetic energy which is possible is zero;

$$E_f = K_f + U_f$$

$$= 0 + 0 = 0$$

Equating initial and final energies, we have

$$E_i = E_f$$

or

$$\frac{1}{2} mv_o^2 - \frac{GM_s m}{r} = 0$$

or

$$v_o^2 = \frac{2GM_s}{r}$$

Substituting numerical values, we obtain

$$vo^2 = \frac{2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{1.5 \times 10^{11}} = 17.8 \times 10^8$$

or $v_o = 4.2 \times 10^4$ m/sec = 42 km/sec

TRUE OR FALSE? The expression $E_i = K_i + U_i$ is a symbolic statement of the principle of conservation of energy.

[a] CORRECT ANSWER: B

The potential energy due to a pair of point masses M and m is given by

$$U = -\frac{GM_m}{r}$$

where r is the distance between the particles. Adding the potential energies due to all pairs of particles in both initial and final configurations, we have

INITIAL (Spaceship at infinity)

$$U_i = -G\left[\frac{M_e M_m}{r_{em}} + \frac{M_s M_e}{r_{se}} + \frac{M_s M_m}{r_{sm}}\right]$$

FINAL (Spaceship at P)

$$U_f = -G\left[\frac{mM_s}{r_{sp}} + \frac{2mM_e}{r_{em}} + \frac{2mM_m}{r_{em}}\right]$$

The work done equals the change in potential

$$W = U_f - U_i = -Gm\left[\frac{M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{2M_m}{r_{em}}\right]$$

TRUE OR FALSE? When the spaceship is at infinity, the potential energy of the entire system is zero.

[b] CORRECT ANSWER: D

Since the gravitational forces are conservative, we can utilize the principle of conservation of energy to solve this problem. If we assume the kinetic energy of m_2 to remain unchanged, then all the work done "against" gravity must go into the potential energy of m_2. Thus,

$$W = \Delta U = U_f - U_i$$

But $U_i = 0$ because initially the particles are infinitely far apart. We have

$$W = U_f = -\frac{Gm_1m_2}{r_{12}} = -\frac{Gm_1m_2}{[(a\sqrt{3})^2 + a^2]^{1/2}}$$

$$= -\frac{Gm_1m_2}{2a}$$

[a] CORRECT ANSWER: $V(r) = -4\sqrt{2}\,\frac{Gm}{a}$

The gravitational potential at a distance r from a point mass m is

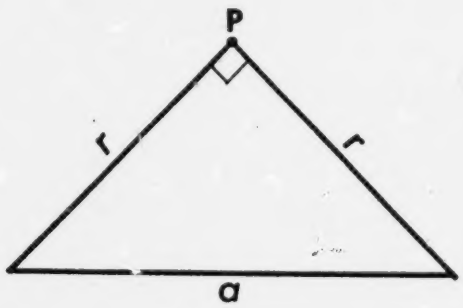
$$V(r) = -\frac{Gm}{r} \qquad (1)$$

Since potential is a scalar quantity (as opposed to the field, which is a vector), the potential at P is the algebraic sum of potentials due to all mass objects; that is,

$$V(r) = -4\,\frac{Gm}{r} \qquad (2)$$

From geometry,

$$r = \frac{a}{\sqrt{2}}$$



Therefore,

$$V(4) = -4\sqrt{2}\,\frac{Gm}{a} \qquad (3)$$

You may notice that the field of the point P is zero; however, the potential is not zero as is evident from equation (3).

TRUE OR FALSE? It is quite possible to have a gravitational potential of 5 nt/kg at a given point in space while the field at the same point is zero.

[b] CORRECT ANSWER: A

Since all other masses in the universe are neglected, there can be no gravitational force on mass m_1. Therefore, the work done by (or against) gravity is zero.

[a] CORRECT ANSWER: B

Potential energy is a scalar quantity so we can compute the potential energy of m_3 due to the presence of m_1 and m_2, respectively, and then add them up (scalarly) to find the total potential energy of m_3. Therefore, find the work done (the change in potential energy) in bringing m_3 to the position (-a, 0):

Change in potential energy due to particle one;

$$U_{31} = -\frac{Gm_1m_3}{r_{31}} = -\frac{Gm_1m_3}{2a}$$

Change in potential energy due to particle two;

$$U_{32} = -\frac{Gm_2m_3}{r_{32}} = -\frac{Gm_2m_3}{(a\sqrt{3})^2 + a^2\ 1/2} = -\frac{Gm_2m_3}{2a}$$

Adding, we obtain total work done

$$= U_{31} + U_{32} = -\frac{Gm_3}{2a}(m_1 + m_2)$$

[b] CORRECT ANSWER: B

The gravitational potential at a distance r from a spherically symmetric body of mass M is

$$V(r) = -GM/r$$

Potential is a *scalar* field (as opposed to field strength, which is a vector field), so the potential at P is the algebraic sum of the contributions of the Earth, the moon and the sun; that is,

$$V = -\frac{GM_s}{r_{sp}} - \frac{GM_e}{r_{ep}} - \frac{GM_m}{r_{mp}}$$

$$= -G\left[\frac{M_s}{r_{sp}} + \frac{M_e}{r_{em}/2} + \frac{M_m}{r_{em}/2}\right] = -G\left[\frac{M_s}{r_{sp}} + \frac{2M_e}{r_{em}} + \frac{2M_m}{r_{em}}\right]$$

The contributions of the Earth and the moon are numerically negligible in comparison to that of the sun. Although P is much closer to the Earth and the moon, the sun is much more massive than the Earth and the moon.

TRUE OR FALSE? The potential V at point P is the vector sum of the potential contributions of the sun, the Earth, and the moon.

[a] CORRECT ANSWER: C

The centripetal force on the satellite is the gravitational force of the Earth. Hence we may write

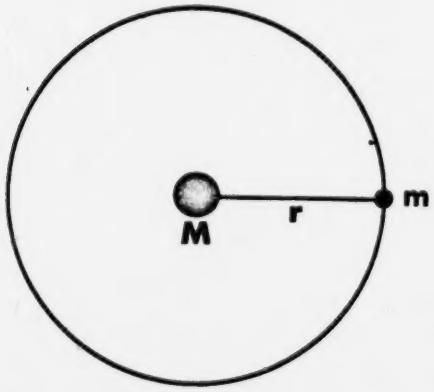
$$\frac{GMm}{r^2} = \frac{mv^2}{r} \qquad (1)$$

or

$$v^2 = \frac{GM}{r} \qquad (2)$$

From this we obtain the satellite's kinetic energy,

$$K = \frac{1}{2} mv^2 = \frac{1}{2} \frac{GMm}{r} \qquad (3)$$



But we know that the potential energy of the system is

$$U = -\frac{GMm}{r} \qquad (4)$$

comparing (4) with (3) we get

$$K = -(1/2)U$$

and

$$E = K + U = -(1/2)U + U = (1/2)U$$

[a] CORRECT ANSWER: C

Since the gravitational force is conservative, the mechanical energy is conserved. The initial separation of the apples is r_i = 1 m and their initial speed is zero so

$$E_i = U_i + K_i = -G\frac{m^2}{r_i} \qquad (1)$$

At the time of collision the separation of the apples' centers is equal to their diameter, $r_f = 2 \times 5$ cm = 10 cm = 0.1 m. Since the apples are identical, each moves with the same speed v_f at the time of collision, thus

$$E_f = U_f + K_f = -G\frac{m^2}{r_f} + 2\left[\frac{1}{2}mv_f^2\right] \qquad (2)$$

Equating E_i to E_f we obtain

$$v_f^2 = Gm\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

or

$$v_f = \left[Gm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right]^{1/2}$$

Substituting numbers in this equation we find

$$v_f = \sqrt{6.67 \times 10^{-11} \times 0.2\left[\frac{1}{0.1} - \frac{1}{1}\right]} = \sqrt{13.3 \times 10^{-12} \times 9}$$

$$= 1.1 \times 10^{-5} \text{ m/sec}$$

[a] CORRECT ANSWER: $v_o = \left[2G\left(\frac{M_e}{R_e} + \frac{M_m}{d - R_e} - \frac{M_e}{x} - \frac{M_m}{d - x}\right)\right]^{1/2}$

Since the requirement in the problem is that the rocket must *just* reach point P, its speed (and hence kinetic energy) must be zero there. The potential energy at P is

$$U = - \frac{GM_e m}{x} - \frac{GM_m m}{d - x} \qquad (1)$$

The rocket must leave the Earth's surface with a total energy equal to U. The initial energy of the rocket consists of its kinetic energy $(1/2)\ mv_o^2$ and its potential energy resulting from the presence of the Earth and of the moon. Thus

$$U = mv_o^2 - \frac{GM_e m}{R_e} - \frac{GM_m m}{(d - R_e)} \qquad (2)$$

However, we have $M_e \approx 80\ M_m$ and $d \approx 60\ R_e$, so the last term in (2) is about 4800 smaller than the term preceding it and it can be neglected. Rearranging the remaining terms and using (1), we obtain

$$v_o^2 = 2G\left[\left(\frac{M_e}{R_e} + \frac{M_m}{d - R_e} - \frac{M_e}{x} - \frac{M_m}{d - x}\right)\right]^{1/2}$$

TRUE OR FALSE? In this solution, the radius of the moon does not enter into the calculations.

[a] CORRECT ANSWER: b

When potential energy was first introduced, we met the one-dimensional equations

$$U(x) - U(x_o) = -\int_{x_o}^{x} F(x)\, dx \tag{1}$$

and

$$F(x) = -\frac{d}{dx} U(x) \tag{2}$$

At that time we saw that in the case of spherical symmetry, we could replace x by r; i.e.,

$$U(r) - U(r_o) = -\int_{r_o}^{r} F(r)\, dr \tag{3}$$

and

$$F(r) = -\frac{d}{dr} U(r) \tag{4}$$

For the gravitational field we choose $r_o = \infty$ with $U(\infty) = 0$. We simply divide equations (3) and (4) each by the mass m of a test particle, to obtain

$$\frac{U(r)}{m} = -\int_{\infty}^{r} \frac{F(r)}{m}\, dr \tag{5}$$

and

$$\frac{F(r)}{m} = -\frac{d}{dr}\ \frac{U(r)}{m} \tag{6}$$

Using the definitions of $\vec{\gamma}$ and V,

$$\vec{\gamma} = \frac{\vec{F}}{m} \quad \text{and} \quad V = \frac{U}{m}$$

we reduce equations (5) and (6) to

$$V(r) = -\int_{\infty}^{r} \gamma(r)\, dr \tag{7}$$

and

$$\gamma(r) = -\frac{d}{dr} V(r) \tag{8}$$

next page

continued

Note that the potential is a scalar quantity, whereas the gravitational field is a vector quantity written in scalar form. Thus, equation (8) represents the vector equation

$$\vec{\gamma}(r) = -\frac{d}{dr} V(r)\, \hat{r} = -\frac{d}{dr} V(r)\, \frac{\vec{r}}{r}$$

[a] CORRECT ANSWER: $v = 2\sqrt{\frac{GM}{L}}$

In this problem the stipulation is that the speed of spaceship should be zero at the point z: to obtain this required solution we equate the mechanical energy of the spaceship at point P to that at point z. Symbolically,

$$U_P + (KE)_P = U_z + (KE)_z \qquad (1)$$

However, the potential energy of a body of mass m, a distance r from a spherically symmetric body of mass M is

$$U(r) = -\frac{GM_m}{r}$$

Therefore, the potential energy at point P is

$$U_P = -\frac{GM_m}{L/2} - \frac{GM_m}{L/2} = -\frac{4GM_m}{L} \qquad (2)$$

and at point z is

$$U_z = -\frac{GM_m}{L} - \frac{GM_m}{L} = -\frac{2GM_m}{L} \qquad (3)$$

Now, the expressions for kinetic energy at those points are

$$(KE)_P = \frac{1}{2} mv^2 \text{ and } (KE)_z = 0 \qquad (4)$$

Since at the point z, the kinetic energy is zero by the requirement of the problem. Substitution of equations (2), (3) and (4) in equation (1) leads to

$$-\frac{4GM_m}{L} + \frac{1}{2} mv^2 = -\frac{2GM_m}{L}$$

next page

continued

Solving for v, we get

$$v = 2\sqrt{\frac{GM}{L}}$$

TRUE OR FALSE? At point P, the spaceship is half as far from the center of either planet as it is when it is at point z.